# Advanced Topics with Extrema in 2 Variables

## Extrema in 2 Variables

**Questions**:

Find and classify the critical points of the following functions:

**1)** $f(x,y)=1+2xy-x^2-y^2$

**2)** $f(x,y)=4-\sqrt{x^2+y^2}$

**3)** $z^3+z+xy-2x-y+2=0, \left[z=f(x,y)\right]$

**4)** $f(x,y)=x^3-y^3-3x^2+6y^2+3x-12y+8$

**5)** $f(x,y,z)=x+\frac{y^2}{4x}+\frac{z^2}{y}+\frac{2}{z},\ (x,y,z>0)$

**6)** Find the least distance between the parabolas $y=x^2+1$ and $y=-x^2+2x$.
Note: this exercise assumes knowledge of numerical methods for solving equations such as the Newton-Raphson Method.

**7)** In each of the following sections, find a curve of the form $y=h(x)$ which minimizes the sum of squared errors (SSE) for the data points given in the form $(x_1,y_1),\ (x_2,y_2),...,(x_n,y)_n$.
In this question, use the SSE function $f(a,b)=\sum_{i=1}^{n}\left(h(x_i)-y_i\right)^2$ and assume that the critical point $(a,b)$ obtained from solving the equations $f_a=0,\ f_b=0$.

a. $h(x)=ax+b$ for the points: $(2,2.5),\ (1,0.8),\ (3,3.2),(4,3.5)$

b. $h(x)=ax^2+bx$ for the points: $(-1,2),\ (2,0),\ (0,-2)$

c. $h(x)=ax+\frac{b}{x}$ for the points: $(10,20.2),\ (6,12.9),\ (4,8.5),\ (0.5,4)$

d. $h(x)=ax^2+b/x^2$ for the points: $(4,33),\ (2,8.5),\ (0.5,2.3),\ (1,4.5),\ (0.1,90)$

e. $h(x)=ax^2+bx+c$ for the points: $(1,4.5),\ (0.5,2.3),\ (0,0.8),\ (-1,0.1),\ (-0.5,0.12)$

## Final answers:

**1)** $(t,t)$ For any real number $t$; Maximum.

**2)** $(0,0)$ ; Maximum.

**3)** $(1,2)$ ; Saddle.

**4)** $(1,2)$ ; Saddle.

**5)** $(0.5,1,1)$ ; Minimum.

**6)** 0.375.

**7)** a. $y=0.88x+0.3$ b. $y=\frac{2}{3}x^2-\frac{4}{3}x$ c. $y=2.032x+\frac{1.5039}{x}$

d. $y=2.06x^2+\frac{0.9}{x^2}$ e. $y=1.48x^2+2.196x+0.824$.