proprep™

Workbook

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Chain Rule

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Questions:

In the following exercises, assume that all given derivatives exist.

1) Given $\ln\left(x^2-y^2\right)$, $x=2u-v$, $y=u^2+v^3$. Compute: z_u , z_v .

2) Given $v=4t+k^2$, $u=t^2+4m$, $z=e^{u-v}$. Compute: $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial m}$, $\frac{\partial z}{\partial k}$.

3) Given $z=f\left(x^2-y^2\right)$. Prove $y\cdot z_x+x\cdot z_y=0$.

4) Given $z=f\left(xy\right)$. Prove $x\cdot z_x-y\cdot z_y=0$.

5) Given $z=f\left(\frac{x}{y}\right)$. Prove $x\cdot z_x+y\cdot z_y=0$.

6) Given $z=f\left(x-y,y-x\right)$. Prove $z_x+z_y=0$.

7) Given $w=f\left(x-y,y-z,z-x\right)$. Prove $w_x+w_y+w_z=0$.

8) Given $u=\sin x+f\left(\sin y-\sin x\right)$. Prove $u_x\cos y+u_y\cos x=\cos x\cos y$.

9) Given $z=y\cdot f\left(x^2-y^2\right)$. Prove $\frac{1}{x}z_x+\frac{1}{y}z_y=\frac{z}{y^2}$.

10) Given $z=xy+xf\left(\frac{y}{x}\right)$. Prove $x\cdot z_x+y\cdot z_y=xy+z$.

11) Given $u\left(x,y,z\right)=x^2\cdot f\left(\frac{y}{x},\frac{z}{x}\right)$. Prove $xu_x+yu_y+zu_z=2u$.

12) Given $h\left(x,y\right)=f\left(y+ax\right)+g\left(y-ax\right)$. Prove $h_{xx}=a^2\cdot h_{yy}$.

13) Given $u\left(x,y\right)=f\left(e^x\sin y\right)-g\left(e^x\sin y\right)$. Prove:

a. $u_{xx}+u_{yy}=\frac{u_{xx}-u_x}{\sin^2 y}$.

b. $u_{xy}=u_{yx}$.

c. Compute: $u_{xy}\left(1,\pi\right)$, given that $f'\left(0\right)=2,\ g'\left(0\right)=1$.

14) Given $u=f(x,y),\ \ x=r\cos\theta,\ y=r\sin\theta$. Prove:

a. $(u_x)^2+(u_y)^2=(u_r)^2+\frac{1}{r^2}(u_\theta)^2$.

b. $u_{rr}=f_{xx}\cos^2\theta+2f_{xy}\cos\theta\sin\theta+f_{yy}\sin^2\theta$.

15) Given $z=h(u,v)$ and that $u=f(x,y)$, $v=g(x,y)$ satisfy the Cauchy-Riemann equations, i.e. $u_x=v_y$, $u_y=-v_x$. Prove:

a. u,v satisfy the Laplace equation, i.e. $u_{xx}+u_{yy}=0$, $v_{xx}+v_{yy}=0$.

b. $h_{xx}+h_{yy}=\left((u_x)^2+(v_x)^2\right)(h_{uu}+h_{vv})$.

16) Given $u=f(x,y),\ \ x=r\cosh s,\ y=r\sinh s$. Prove: $(u_x)^2-(u_y)^2=(u_r)^2-\frac{1}{r^2}(u_s)^2$.

Answer Key:

1) $z_u=\frac{2\cdot 2(2u-v)}{(2u-v)^2-(u^2+v^3)^2}-\frac{2\cdot 2(u^2+v^3)u}{(2u-v)^2-(u^2+v^3)^2}$, $z_v=\frac{2x}{x^2-y^2}\cdot(-1)+\frac{-2y}{x^2-y^2}3v^2$

2) $\frac{\partial z}{\partial t}=2e^{t^2-4t+4m-k^2}(t-2)$, $\frac{\partial z}{\partial m}=e^{u-v}\cdot 4$, $\frac{\partial z}{\partial k}=-e^{u-v}2k$

3-12) Refer to the videos.

13) a+b. Refer to the videos. c. $-e$

14-16) Refer to the videos.