proprep

Workbook

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Number Representation

Number System

Questions

1) Convert the following numbers from their base into a decimal base:

a. $(23)_5$

b. $(516)_7$

c. $(12212)_3$

d. $(82)_{12}$

e. $(41.23)_5$

f. $(104.33)_6$

2) Convert the following numbers from their octal base into a decimal base:

a. $(415)_8$

b. $(127)_8$

c. $(51.6)_8$

d. $(64.04)_8$

3) Convert the following numbers from their binary base into a decimal base:

a. $(10110)_2$

b. $(110110101)_2$

c. $(0.1011)_2$

d. $(11011.00101)_2$

4) Convert the following numbers from their hexadecimal base into a decimal base:

a. $(8A)_{16}$

b. $(6B5)_{16}$

c. $(49.F)_{16}$

d. $(C.1D)_{16}$

Answer Key

1) a. $(13)_{10}$ b. $(285)_{10}$ c. $(158)_{10}$ d. $(98)_{10}$ e. $(21.52)_{10}$ f. $\left(40\frac{7}{12}\right)_{10}$

2) a. $(269)_{10}$ b. $(87)_{10}$ c. $(41.75)_{10}$ d. $\left(52\frac{1}{16}\right)_{10}$

3) a. $(22)_{10}$ b. $(437)_{10}$ c. $\left(\frac{4}{16}\right)_{10} = (0.6875)_{10}$ d. $\left(27\frac{5}{32}\right)_{10} = (27.15625)_{10}$

4) a. $(138)_{10}$ b. $(1717)_{10}$ c. $\left(73\frac{15}{16}\right)_{10}$ d. $\left(12\frac{29}{256}\right)_{10}$

Arithmetic Operations between Numbers in Various Bases

Questions

1) Given the two binary numbers A and B, find the sum A+B and the difference A-B:

a. $A=(210)_3$, $B=(122)_3$

b. $A=(573)_9$, $B=(161)_9$

c. $A=(360)_8$, $B=(21)_8$

d. $A=(FF20)_{16}$, $B=(E40A)_{16}$

2) Find the sum and the difference in each case:

a. $A=(101101)_2$, $B=(1011)_2$; $A+B=?$; $A-B=?$

b. $A=(11111111)_2$, $B=(100001)_2$; $A+B=?$; $A-B=?$

3) Using long multiplication, calculate the following:

a. $(750)_8 \cdot (62)_8$

b. $(66)_7 \cdot (55)_7$

c. $(AB)_{16} \cdot (E7)_{16}$

d. $(A0)_{12} \cdot (12)_{12}$

4) Using long multiplication, calculate the following:

a. $(100010)_2 \cdot (110)_2$

b. $(101110)_2 \cdot (1011)_2$

5) Given two numbers A and B where: $A=(a_{n-1}a_{n-2}...a_0)_2$ and $B=(b_{m-1}b_{m-2}....b_0)_2$, prove that the result of the multiplication $A \cdot B$ will include $n+m$ bits.

6) Calculate the following divisions:

a. $(110100)_2 : (100)_2$

b. $(1111000)_2 : (1100)_2$

c. $(1100100)_2 : (1101)_2$

7) Denoting: $A=(101)_2 \cdot (110)_2$ and $B=(1010)_2 \cdot (1x)_2$ where x is an unknown bit. Given that $A-B=0$, find x.

Answer Key

1) a. $A+B=(1102)_3$, $A-B=(011)_3$ b. $A+B=(744)_9$, $A-B=(412)_9$

c. $A+B=(401)_8$, $A-B=(337)_8$ d. $A+B=(1E23A)_{16}$, $A-B=(1B16)_{16}$

2) a. $A+B=(111000)_2$, $A-B=100010$ b. $A+B=100100000$, $A-B=11011110$

3) a. $(57520)_8$ b. $(5412)_7$

c. $(9\text{D}4\text{A})_{16}$ d. $(1000)_{12}$

4) a. $(11001100)_2$ b. $(111111010)_2$

5) $R=m+n$

6) a. $(1101)_2$ b. $(1010)_2$ c. $\left(111\frac{1001}{1101}\right)_2$

7) $X=1$

Conversions between Bases

Questions

1) Convert the following decimal numbers into binary base:

a. 38 b. 244 c. 321

2) Convert the following decimal numbers into binary base:

a. 0.125 b. 0. 84375

c. 0. 33 d. 0.833

3) Convert the following decimal numbers into binary base:

a. 12.25 b. 32.375

4) Write the number $(61.25)_{10}$ in base 3 and in base 8.

5) Write the following binary numbers in octal base:

a. $(101)_2$ b. $(1011)_2$

c. $(10010101)_2$ d. $(11011101000101101101)_2$

e. $(101.1)_2$ f. $(100101.0010111)_2$

6) Write the following binary numbers in hexadecimal base:

a. $(101)_2$ b. $(1011)_2$

c. $(10010101)_2$ d. $(11011101000101101101)_2$

e. $(101.1)_2$ f. $(100101.0010111)_2$

7) Write the following numbers in binary base:

a. $(FB)_{16}$ b. $(40A)_{16}$ c. $(9.9)_{16}$

d. $(66)_8$ e. $(702)_8$ f. $(2.32)_8$

8) Convert the decimal number 0.714 into binary base, octal base and hexadecimal base, with accuracy of at least 6 digits.

9) Write the following numbers in the required base:

a. The number $(53)_6$ in base 9.

b. The number $(104)_7$ in base 5.

c. The number $(0.02)_3$ in base 6.

d. The number $(32.13)_4$ in base 8.

10) Consider the number 0.02 in base 3.

a. Write the number in base 5.

b. What is the number's digit period?

c. How many digits will be in the number (in base 5), for accuracy of less than 10^{-3}?

d. What is the number's accuracy when it is written with to ten decimal places?

11) Find the base in which each equation is true:

a. $(101)^2 + 2004 = 12210$

b. $3 \cdot 5 + 24 = 51$

c. $\dfrac{40 - 7 \cdot 2}{6} = 3$

12) Find the base in which each equation is true:

a. $\sqrt{301} = 13$

b. $\sqrt[4]{121} = 3$

13) Find all of the bases in which each inequality is valid:

a. $(11)^2 < 122$

b. $(11)^3 < 1451$

14) Consider the following equation: $(22)_x + (22)_y = (102)_x$,

where x and y are two different bases ($x, y \in \mathbb{N}$).

a. Find the relation between x and y.

b. Given that the difference between the bases is 1, find x and y.

Answer Key

1) a. $(100110)_2$ b. $(11110100)_2$ c. $(101000001)_2$

2) a. $(0.001)_2$ b. $(0.11011)_2$ c. $(0.0101010001\ldots)_2$ d. $(0.11010101\ldots)_2$

3) a. $(1100.01)_2$ b. $(100000.011)_2$

4) $(61.25)_{10} = (2021.0202)_3$, $(75.2)_8$

5) a. $(5)_8$ b. $(13)_8$ c. $(225)_8$ d. $(3350555)_8$
e. $(5.4)_8$ f. $(45.134)_8$

6) a. $(5)_{16}$ b. $(13)_{16}$ c. $(95)_{16}$ d. $(DD16D)_{16}$
e. $(5.8)_{16}$ f. $(25.2E)_{16}$

7) a. $(11111011)_2$ b. $(010000001010)_2$ c. $(1001.1001)_2$ d. $(110110)_2$
e. $(111000010)_2$ f. $(10.01101)_2$

8) $(0.101101)_2 = (0.B4)_{16}$

9) a. $(36)_9$ b. $(203)_5$ c. $(0.12)_6$ d. $(16.34)_8$

10) a. $(102342)_5$ b. 102342 period c. $(10234)_5$ d. $9.1 \cdot 10^{-8}$

11) a. $X = 5$ b. $X = 6$ c. $X = 8$

12) a. $X = 4$ b. $X = 8$

13) a. $r \geq 3$ b. $r \geq 6$

14) a. $X = 1 + \sqrt{2y+3}$ b. $X = 4$, $y = 3$

Complementary Numbers

Questions

1) Find the complementary numbers as requested in each case:

a. The 9's complement of $(8054)_{10}$.

b. The 9's complement of $(0.625)_{10}$.

c. The 10's complement of $(8054)_{10}$.

d. The 10's complement of $(8054)_{10}$.

2) Find the complementary numbers as requested in each case:

a. The 1's complement of $(110101)_2$.

b. The 1's complement of $(0.00011)_2$.

c. The 1's complement of $(110101)_2$.

d. The 1's complement of $(0.00011)_2$.

3) Consider the number $(5AF3)_{16}$.

a. Find the 16's complement of the given number.

b. Convert the number into binary representation and find its 2's complement.

c. Compare the results from parts a and b.

Answer Key

1) a. 1,945 b. 9.374 c. 1,946 d. 9.375

2) a. $(001010)_2$ b. $(1.111)_2$ c. $(1011)_2$ d. $(1.11101)_2$

3) a. $(A50D)_{16}$ b. $(1010010100001101)_{2's}$ c. Proof

Subtraction using Complementary Numbers

Questions

1) Subtract the following numbers:

a. $(3752)_{10} - (748)_{10}$ using 10's complement.

b. $(352)_{10} - (6408)_{10}$ using 10's complement.

c. $(3752)_{10} - (748)_{10}$ using 9's complement.

d. $(352)_{10} - (6408)_{10}$ using 9's complement.

2) Subtract the following numbers:

a. $(10110110)_2 - (10010)_2$ using 2's complement.

b. $(10010)_2 - (11000011)_2$ using 2's complement.

c. $(10110110)_2 - (10010)_2$ using 1's complement.

d. $(10010)_2 - (11000011)_2$ using 1's complement.

Answer Key

1) a. $(3004)_{10}$ b. $(-6056)_{10}$ c. 3004 d. $(-6056)_{10}$

2) a. 10100110 b. $(-10110001)_2$ c. $(10100100)_2$ d. $(-10110001)_2$

Signed Binary Numbers and Arithmetic Operations

Questions

1) Represent the following numbers in Sign-Magnitude (SM), 1's complement, 2's complement.

a. -43
b. -75
c. -158

2) Given two numbers in Sign-Magnitude (SM), (+863) and (+3562), calculate:

a. 3562 + 863
b. 3562 – 863
c. -3562 + 863
d. -3562 – 863

3) Given the numbers 73 and 42, convert them into binary representation and calculate the following in a binary base (using 2's complement):

a. (+42) + (-73)
b. (-42) + (-73)

4) Given $a = (01000101.0011)_2$ and $b = (10110)_2$ which are represented in $r-1$'s complement, find the value of $\frac{a}{b}$ (please also record in $r-1$'s complement).

5) Given $a = (1010010.1)_2$ and $b = (1000)_2$ which are represented in r's complement, find the value of $\frac{a}{b}$ (please also record in r's complement).

Answer Key

1)

	SM	1's	2's
a.	10101011	11010100	11010101
b.	11001011	10110100	10110101
c.	110011110	101100001	101100010

2) a. 04452 b. 02699 c. 97301 d. 95575

3) a. $(-31)_{10}$ b. $(-115)_{10}$

4) $(1000.01)_2$

5) $(0101.1011)_2$

Binary Codes

Questions

1) Represent the number 5401 according to each of the following binary codes:

a. BCD
b. Excess-3
c. 8,4,-2,-1
d. 2,4,2,1

2) Write down the numbers 423 and 629 in BCD representation and perform addition according to the code.

3) A 4-digit base has the weights 6,3,1,1.
Write down a truth table for the 10 decimal digits in this base.

4) A binary code has 10 bits which represent each of the 10 decimal digits.
Each digit has been represented with 9 zeros and one value of 1.

a. Suggest 2 different ways to represent the 10 decimal digits using this type of code.
b. A new code has been offered, which will also represent the 10 decimal digits, but with 5 bits as follows:
   - For even numbers the code will include 4 zeros and 1 one at the location of half of the decimal digit's value.
   - For odd numbers the code will include 4 ones and 1 zero in the location of half of the next digit of the decimal number.
   i. Which decimal digit has been represented by the code 01000?
   ii. Which digit has been represented by the code 10111?
   iii. Write the code for digits 4 and 5.

5) Write ASCII code to the phrase "Hello World" using 8-bit representation and one parity bit.

6) A bi-quinary code was widely used in computers in the past.
This code contains 7 bits according to the weights 5043210.

a. Write a truth table for the 10 decimal digits using this code and find what is "special" about it.
b. Calculate the sum 1000100 + 0100100 and write the result using this code.
c. Can adding a parity bit help for this code?
d. What can you say about the way of representing digits using this code?

7) A common method for coding information in communication systems is called m-out-of-n, where m and n are been chosen such that they will allow 10 different combinations.
One commonly used type of code from this family is 2-out-of-5.
In this method, we can represent the 10 digits according to the weights 01236.
As a ground rule, the number 0 will be represented as 01100.

a. Write a truth table for the 10 decimal digits using 2-out-of-5 code.
b. Write the number 11000 01100 00110 00011 as a decimal.

Answer Key

1) Normal: $(1010100011001)_2$

a. 0101 0100 0000 0001 b. 1000 0111 0011 0100

c. 1011 0100 0000 0111 d. 1011 0100 0000 0001

2) $(1052)_{10}$

3) Table:

	6 3 1 1
0	0000
1	0001
2	0011
3	0100
4	0110
5	0111
6	1000
7	1001
8	1011
9	1111

4) a. Proof

b. i. 6 b. ii. 7 b. iii. 4 = 00100 , 5 = 11011

5) See table in the video solution.

6) a. See truth table in the video solution. b. $[1010000]_{Bq}$ c. No

7) a. See truth table in the video solution. b. $(1059)_{10}$