proprep™

Workbook

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Constrained Extrema

Practice Questions

Questions:

1) Find: $\max\{\ln x+\ln y\}$ $s.t.$ $y+2x=8$ and show that the maximum is $\ln 8$.

2) Find: $\min\{x+y\}$ $s.t.$ $x+y+xy=15$ where $x\geq 0$, $y\geq 0$ and show that the minimum is 6.

3) Find the extrema of the function $f(x,y)=4x+6y$ subject to the constraint $x^2+y^2=13$.

4) Find the extrema of the function $f(x,y)=x^2y$ subject to the constraint $x^2+2y^2=6$.

5) Answer the following questions:

a. Solve the extremum problem $\max\{xy\}$ $s.t.$ $x+3y=12$, $x,y>0$.

b. Interpret the solution graphically.

6) Answer the following questions:

a. Solve the extremum problem $\min\{2x+y\}$ $s.t.$ $\sqrt{x}+\sqrt{y}=9$, $x,y>0$.

b. Interpret the solution graphically.

7) From all points on the line $x+3y=12$, find the one with the greatest product of its coordinates.

8) From all points on the curve $2x^2+3xy=1-2y^2$, find the ones with the greatest and shortest distances from the origin.

9) Find the shortest distance from the line $3x-6y+4=0$ to the parabola $x^2+2xy+y^2+4y=0$.

10) From all the open boxes whose volume is 32cm^3, compute the dimensions of the one with least surface area.

11) Find the points on the sphere $x^2+y^2+z^2=36$ which are, respectively, closest to and furthest from the point $(1,2,2)$.

12) Answer the following questions:

a. Find the shortest distance from the point $(1,2,3)$ to the plane $-2x-2y+z=0$.

b. Find the point on the plane $-2x-2y+z=0$ closest to the point $(1,2,3)$.

c. Check your answer using the formula for the distance from a point to a plane.

13) Find the point(s) on the surface $z^2 = xy+1$ closest to the origin.

14) Find the greatest and the least distances, respectively, from the ellipsoid $\frac{x^2}{96}+y^2+z^2=1$ to the plane $3x+4y+12z=288$.

15) Find the minimum and the maximum distances from the origin of the curve obtained by intersecting the cylinder $x^2+y^2=1$ and the plane $z=x+y$.

16) Find the minimum and the maximum distances from the origin of the curve obtained by intersecting the ellipsoid $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and the plane $z=x+y$.

17) In the market, Joe buys x kg of cucumbers and y kg of tomatoes. The utility function is given by $u(x,y)=\ln x+\ln y$. Cucumbers cost $\$1$ per kg and tomatoes cost $\$2$ per kg. Joe's goal is to achieve a utility level of $\ln 32$ and he'd like to reach it at the least possible cost.Formulate (mathematically) and solve Joe's problem.

18) The transformation curve between x mangos and y pineapples is $x^2+y^2=13$.
The utility function is $u(x,y)=4x+6y$.
Jane is looking for the basket $(x,y)=$ (mangoes, pineapples) on the transformation curve which maximizes her utility from consuming the fruit. Formulate (mathematically) and solve Jane's problem.

19) In the market, Danny buys x *kgs* of cucumbers and y *kgs* of tomatoes.
The utility function is given by $u(x,y)=xy$.
The price of cucumbers is $1 per *kg* and the price of tomatoes is $3 per *kg*.
Danny has a budget of $12. Formulate (mathematically) and solve Danny's problem.

20) A manufacturer has a production function $Q=\sqrt{K}+\sqrt{L}$ where Q is output, K is capital, and L is labor. The unit prices for K and L are: $P_K=2$, $P_L=1$. The manufacturer is at an output level of 100 and is looking for the combination (K^*,L^*) which minimizes cost. Formulate (but don't solve) the manufacturer's problem mathematically.

Answer Key:

1) $(2,4)$

2) $(3,3)$

3) Max $A(2,3)$, Min $B(-2,-3)$ under constraint.

4) Max: $A(2,1)$, $B(-2,1)$, min: $C(-2,-1)$, $D(2,-1)$ under constraint

5) a. $(6,2)$ value=12 b. graph

6) a. $(9,36)$ value=54 b. graph

7) $(6,2)$

8) Max $\sqrt{2}$, Min $=\sqrt{\frac{2}{7}}$

9) $\frac{3}{\sqrt{45}}$

10) 48cm^2

11) $A(2,4,4)$, $B(-2,-4,-4)$.

12) A.1 b. $\left(\frac{1}{3},\frac{4}{3},\frac{10}{3}\right)$

13) $(0,0,1)$, $(0,0,-1)$.

14) Least distance to the plane: $\left(9,\frac{1}{8},\frac{3}{8}\right)$, greatest distance to the plane: $\left(-9,-\frac{1}{8},-\frac{3}{8}\right)$.

15) Min: 1, max: $\sqrt{3}$.

16) Max: $\sqrt{10}$, min: $\sqrt{\frac{75}{17}}$

17) $\min\{x+2y\}$ $s.t.$ $xy=32$, $(x,y>0)$ Solution: $(8,4)$, cost: $16\$$

18) $\max\{4x+6y\}$ $s.t.$ $x^2+y^2=13$, $(x,y\geq 0)$ Solution: $(2,3)$, utility: 26

19) $\max\{xy\}$ $s.t.$ $x+3y=12$, $(x,y>0)$. Solution: $(6,2)$, cost: $12\$$

20) $\min\{2K+L\}$ $s.t.$ $Q=\sqrt{K}+\sqrt{L}$, $(K,L>0)$