

# Workbook



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# Continuity of a Function

## Definition of Continuity

### Questions

1) Consider the function:  $f(x) = \begin{cases} x & x \geq 1 \\ x^2 & x < 1 \end{cases}$ .

Is the function  $f(x)$  continuous at  $x=1$ ? Sketch the graph of  $f(x)$ .

2) Consider the function:  $f(x) = \begin{cases} x+1 & x \leq 2 \\ 5-x & x > 2 \end{cases}$ .

Is the function  $f(x)$  continuous at  $x=2$ ? Sketch the graph of  $f(x)$ .

3) Is the function  $f(x) = \begin{cases} \frac{\sin 4x}{x} & x > 0 \\ 4 + e^{\frac{1}{x}} & x < 0 \end{cases}$  continuous at  $x=0$ ?

4) Is the function  $f(x) = \begin{cases} \frac{\sin x}{x} & x > 0 \\ 2 & x = 0 \\ 1 + e^{\frac{1}{x}} & x < 0 \end{cases}$  continuous at  $x=0$ ?

5) Is the function  $f(x) = \begin{cases} \sin x & x < 0 \\ x^2 & 0 \leq x < 1 \\ 2-x & 1 \leq x < 2 \\ x-3 & x \geq 2 \end{cases}$  continuous at  $x=0, 1, 2$ ?

6) Is the function  $f(x) = \begin{cases} \frac{1}{x} & x \leq 1 \\ |x-2| & 1 < x < 2 \\ 1 & x = 2 \\ x-2 & x > 2 \end{cases}$  continuous at  $x=1, 2$  ?

7) Find the value of  $k$  for which  $f(x) = \begin{cases} kx^2 + x - 2 & x < 2 \\ 5kx - 6 & x > 2 \end{cases}$  is continuous everywhere.

8) Find  $k$  which makes the function  $f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x - 1} & x \neq 1 \\ k & x = 1 \end{cases}$  continuous for every  $x$ .

9) Find  $k$  which makes the function  $f(x) = \begin{cases} \frac{\sqrt{x^2 + 5} - 3}{x - 2} & x \neq 2 \\ k & x = 2 \end{cases}$  continuous for every  $x$ .

10) Find  $k$  which makes the function  $f(x) = \begin{cases} 2x - k & x \leq 0 \\ 5kx - 6 & x > 0 \end{cases}$  continuous for every  $x$ .

11) Find the values of constants  $a$  and  $b$  for which function  $f(x) = \begin{cases} ax + b & x \leq 0 \\ \frac{\sin x}{2x} & 0 < x < \pi \\ a \cos x & x \geq \pi \end{cases}$

is continuous in its domain.

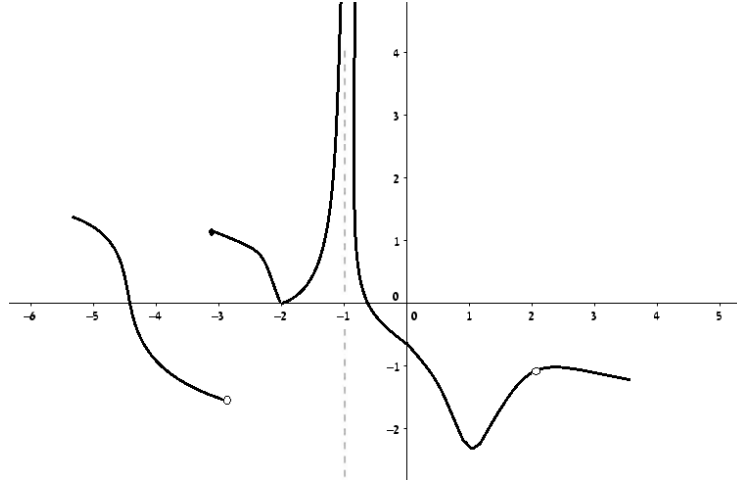
### Answer Key

- 1) Continuous at  $x = 1$
- 2) Continuous at  $x = 2$
- 3) Not defined at  $x = 0$
- 4) Not continuous
- 5) Continuous at  $x = 0$ , Continuous at  $x = 1$ , Not continuous at  $x = 2$
- 6) Continuous at  $x = 1$ , Not continuous at  $x = 2$
- 7)  $k = 1$
- 8)  $k = 4$
- 9)  $k = \frac{2}{3}$
- 10)  $k = -1$
- 11)  $a = 0, b = \frac{1}{2}$

## Points of Discontinuity

### Questions

1) Classify any points of discontinuity of  $f$  over the graphed interval.



2) Find and classify any points of discontinuity of the functions below:

a.  $f(x) = \frac{x+1}{x-1}$

b.  $f(x) = \frac{x^2 + x - 2}{x^2 - 1}$

3) Find and classify any points of discontinuity of the functions below:

a.  $f(x) = \begin{cases} x^2 & x < 1 \\ 0 & x = 1 \\ 2-x & x > 1 \end{cases}$

b.  $f(x) = \begin{cases} x^2 & x < 1 \\ 0 & x = 1 \\ -x^2 + 2x + 1 & x > 1 \end{cases}$

c.  $f(x) = \begin{cases} x^2 & x < 1 \\ 0 & x = 1 \\ \frac{1}{x-1} & x > 1 \end{cases}$

**Answer Key**

- 1)  $x = -3$ ,  $x = -1$ ,  $x = 2$ .
- 2) a.  $x = 1$  undefined                      b.  $f(1)$  undefined
- 3) a.  $x = 1$  removable discontinuity      b. Jump discontinuity at  $x = 1$   
c. Essential discontinuity at  $x = 1$

## The Intermediate Value Theorem

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### Questions:

- 1) Use the intermediate value theorem to show that the equation  $\cos x = x$  must have at least one solution.
- 2) Use the intermediate value theorem to show that the equation  $x^3 + 4x = 1$  must have at least one solution.
- 3) Use the intermediate value theorem to show that the equation  $\ln x = -x^2$  must have at least one solution.
- 4) Use the intermediate value theorem to show that the equation  $x^3 + bx^2 + cx + d = 0$  must have at least one solution.
- 5) Use the intermediate value theorem to show that the equation  $4x^3 + 5x = \frac{1}{x}$  must have at least two solutions.
- 6) Use the intermediate value theorem to show that the equation  $e^x = 5x$  must have at least one solution.



\*Proof questions- for the solution see the video