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Workbook

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Differentiability

Differentiability of Piecewise Functions

Questions

1) Determine points of non-differentiability of the following functions, and find $f'(2)$, if possible.

a. $f(x)=\begin{cases} x^2-4x & x\geq 2 \\ x^3-14 & x<2 \end{cases}$

b. $f(x)=\begin{cases} x^2-5x & x\geq 2 \\ x^3-14 & x<2 \end{cases}$

c. $f(x)=\begin{cases} x^2+8x & x\geq 2 \\ x^3+12 & x<2 \end{cases}$

2) Determine points of non-differentiability of the following functions, and write the formula for $f'(x)$:

a. $f(x)=\begin{cases} \ln(1+2x) & -0.5<x<0 \\ x^2+2x & x\geq 0 \end{cases}$

b. $f(x)=2+4|x-1|$

c. $f(x)=3x^2+x|x|+1$

3) Determine points of non-differentiability of the following functions, and find $f'(0)$ if possible:

a. $f(x)=\begin{cases} x\cdot\sin\frac{1}{x} & x>0 \\ 0 & x\leq 0 \end{cases}$

b. $f(x)=\begin{cases} x^2\cdot\sin\frac{1}{x} & x>0 \\ 0 & x\leq 0 \end{cases}$

4) For what value or values of the constants a and b, is f differentiable for all values of x:

a. $f(x)=\begin{cases} x^2+ax & x\geq 2 \\ x^3+b & x<2 \end{cases}$

b. $f(x)=\begin{cases} e^x & x\leq 0 \\ ax+b & x>0 \end{cases}$

c. $f(x)=\begin{cases} \ln^3 x & x\geq e \\ ax+b & x<e \end{cases}$

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Answer Key

1) a. $x=2$ b. $x=2$ c. $f(x)$ is differentiable for all x.

2) a. $f'(x)=\begin{cases}\dfrac{2}{1+2x} & -0.5<x<0\\ 2 & x=0\\ 2x+2 & x>0\end{cases}$ b. $x=1$

c. $f(x)$ is differentiable for all x, $f'(x)=\begin{cases}8x & x>0\\ 0 & x=0\\ 4x & x<0\end{cases}$.

3) a. $f'(x)=\begin{cases}\sin\dfrac{1}{x}-\dfrac{1}{x^2}x\cos\dfrac{1}{x} & x>0\\ \text{not exist} & x=0\\ 0 & x<0\end{cases}$ b. $f'(x)=\begin{cases}2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x} & x>0\\ 0 & x=0\\ 0 & x<0\end{cases}$.

4) a. $a=8$, $b=12$ b. $b=1$ c. $a=\dfrac{3}{e}$, $b=-2$.

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