proprep™

Workbook

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Divergence Theorem

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Questions

In each of the exercises **1-3** verify the Divergence Theorem $\left(\iiint_R div\mathbf{F}dV = \iint_S \mathbf{F}\cdot\mathbf{n}dS\right)$:

$\mathbf{n}$ is the outward unit normal to S (See remark on notation below).

1) Where $\mathbf{F}=(2x-z)\mathbf{i}+x^2y\mathbf{j}-xz^2\mathbf{k}$ and S is the surface of the cube R determined by the planes: $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$.

2) Where $\mathbf{F}=x\mathbf{i}-2y\mathbf{j}+3z\mathbf{k}$ and S is the surface of the unit ball. $x^2+y^2+z^2\le 1$ given by R

3) Where $\mathbf{F}=(2xy+z)\mathbf{i}+y^2\mathbf{j}-(x+3y)\mathbf{k}$ and S is the surface of the pyramid R determined by the planes: $2x+2y+z=6,\ x=0,\ y=0,\ z=0$.

4) Let S be the surface of the body bounded by the cylinder $x^2+y^2=9$ and the planes $z=0$ and $z=2$. Compute the flux of the vector field $\mathbf{F}=x^3\mathbf{i}+y^3\mathbf{j}+z^2\mathbf{k}$ through S. I.e. compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S.

5) Compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S, $\mathbf{F}=(z^2-x)\mathbf{i}-xy\mathbf{j}+3z\mathbf{k}$, and S is the surface of the body bounded by: $x=0,\ x=3,\ z=4-y^2,\ z=0$.

6) Compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S

$\mathbf{F}=xz^2\mathbf{i}+(x^2y-z^3)xy\mathbf{j}+(2xy+y^2z)\mathbf{k}$, and S is the surface of the body bounded by: $z=\sqrt{a^2-x^2-y^2},\ z=0$.

7) Let S be the open surface $0 \le y \le 4$, $x^2 + z^2 = 16$ (a cylinder without the bases).
Compute the **flux** of the vector field $\mathbf{F} = z^2\mathbf{i} + 5y\mathbf{j} + x^5\mathbf{k}$ through S.
I.e. compute $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ where $\mathbf{n}$ is the outward unit normal to S.

8) Compute $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ where $\mathbf{n}$ is the outward unit normal to S,
$\mathbf{F} = \left(\frac{x^2 y}{1+y^2} + 6yz^2 \right)\mathbf{i} + 2x \arctan y\mathbf{j} - \frac{2xz(1+y)+1+y^2}{1+y^2}\mathbf{k}$, and S is the open surface
$z = 4 - x^2 - y^2, \quad z \ge 0$.

Remark on Notation:

The Divergence Theorem states that given a vector field
$\mathbf{F} = f(x,y,z)\mathbf{i} + g(x,y,z)\mathbf{j} + h(x,y,z)\mathbf{k}$, the equality $\iiint_R div\mathbf{F}dV = \iint_S \mathbf{F} \cdot \mathbf{n} dS$ holds.

There are various other formulations of the Divergence Theorem, such as:

$$\iiint_R \nabla \cdot \mathbf{F} dV = \iint_S \mathbf{F} \cdot \mathbf{n} dS$$

$$\iiint_R (f_x + g_y + h_z) dV = \iint_S \mathbf{F} \cdot \mathbf{n} dS$$

$$\iiint_R (f_x + g_y + h_z) dV = \iint_S fdydz + gdzdx + hdxdy$$

Answer Key

1) The common value is $\frac{11}{6}$

2) The common value is $\frac{8}{3}\pi$

3) The common value is 27

4) 279π

5) 16

6) $\frac{2\pi a^5}{5}$

7) 0

8) -4π