proprep™

# Workbook

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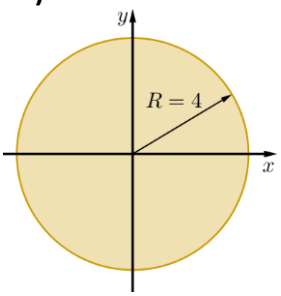
# Double Integrals in Polar Coordinates

## Double Integrals in Polar Coordinates

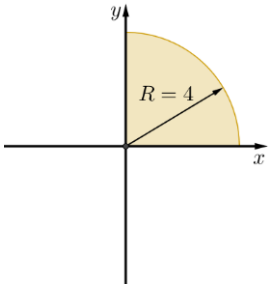
### Questions

Compute: $\iint_D \sqrt{x^2+y^2}\,dA$ where $D$ is the domain described in the sketch.

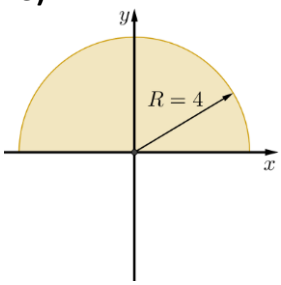
**1)**



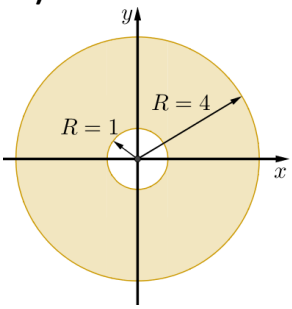

**2)**



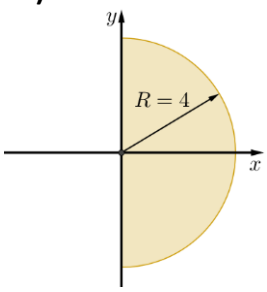

**3)**



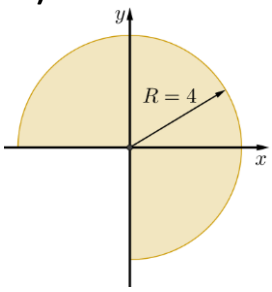

**4)**



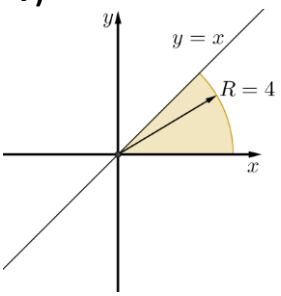

**5)**



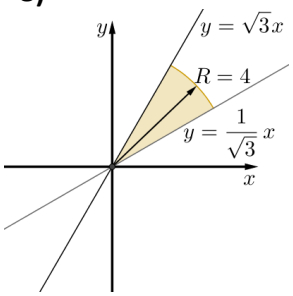

**6)**



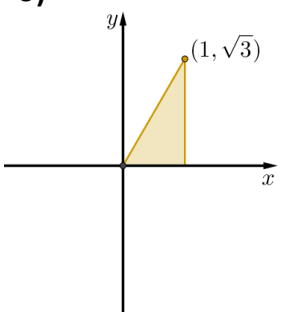

**7)**




**8)**




**9)**



Compute the following integrals by converting to polar coordinates:

**10)** $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$

**11)** $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dydx$

**12)** $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} \left(x^2+y^2\right)dxdy$

**13)** $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \left(x^2+y^2\right)dxdy$

**14)** $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$

**15)** $\int_{0}^{2}\int_{0}^{\sqrt{4-y^2}} \left(x^2+y^2\right)dxdy$

**16)** $\int_{0}^{6}\int_{0}^{y} xdxdy$

**17)** $\int_{0}^{2}\int_{0}^{x} ydydx$

**18)** $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}dydx$

**19)** $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0} \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}dxdy$

**20)** $\int_{0}^{\ln 2}\int_{0}^{\sqrt{\ln^2 2-y^2}} e^{\sqrt{x^2+y^2}}dxdy$

**21)** $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dydx$

**22)** $\int_{0}^{2}\int_{0}^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}dydx$

**23)** $\int_{0}^{2}\int_{-\sqrt{1-(y-1)^2}}^{0} xy^2dxdy$

**24)** $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2+y^2+1\right)dxdy$

**25)** $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{\left(1+x^2+y^2\right)^2}dydx$

In each of the following, compute the volume of the described solid:

**26)** The solid bounded by the sphere $x^2+y^2+z^2=9$ and the cylinder $x^2+y^2=1$.

**27)** The solid bounded inside the cylinder $x^2+y^2=2y$, below the cone $z=\sqrt{x^2+y^2}$, and above the $xy$ plane.

**28)** The solid bounded inside the cylinder $x^2+y^2=x$, below the paraboloid $z=1-x^2-y^2$, and above the $xy$ plane.

## Answer Key

**1)** $\frac{128\pi}{3}$

**2)** $\frac{32\pi}{3}$

**3)** $\frac{64\pi}{3}$

**4)** $42\pi$

**5)** $\frac{64\pi}{3}$

**6)** $32\pi$

**7)** $\frac{16\pi}{3}$

**8)** $\frac{32\pi}{9}$

**9)** $S=\frac{1}{3}\int_0^{\pi/3}\frac{1}{\cos^3\theta}d\theta$

**10)** $\frac{\pi}{2}$

**11)** $\pi$

**12)** $\frac{\pi}{8}$

**13)** $\frac{\pi}{2}$

**14)** $\pi a^2$

**15)** $2\pi$

**16)** 36

**17)** $\frac{4}{3}$

**18)** $\pi\ln\frac{e}{2}$

**19)** $\pi(4-\pi)$

**20)** $\frac{\pi}{2}\ln\frac{4}{e}$

**21)** $\frac{\pi(e-1)}{4e}$

**22)** $\frac{\pi}{2}+1$

**23)** $-\frac{4}{5}$

**24)** $\pi\ln\frac{4}{e}$

**25)** $\pi$

**26)** $\frac{\left(108-64\sqrt{2}\right)\pi}{9}$

**27)** $\frac{32}{9}$

**28)** $\frac{5\pi}{32}$