

Workbook



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Line Integrals

General Questions with Line Integrals

Questions

1) Compute the following:

a. $\int_C (1-x^2) ds$ Where $C: x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$

b. $\int_C x ds$ Where $C: x = t - \sin t, y = 1 - \cos t, 0 \leq t \leq \pi$

c. $\int_C (x+y) ds$ Where C is the line segment joining $O(0,0)$ with $A(1,2)$.

d. $\int_C (x+y^2) ds$ Where C is the perimeter of $\triangle OAB: O(0,0), A(0,1), B(1,0)$

2) Compute the following:

a. $\int_C (x^2 + y^2 + z^2) ds$ where $C: x = \cos t, y = \sin t, z = t, 0 \leq t \leq \pi$.

b. $\int_C (x^3 + 3z) ds$ where $C: x = t, y = \frac{1}{\sqrt{2}}t^2, z = \frac{1}{3}t^3, 0 \leq t \leq 3$.

3) Compute the length of the curve: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.

4) A coil made of thin wire is expressed by $x = \cos t, y = \sin t, z = 2t$ ($0 \leq t \leq \pi$).
Compute the **mass** of the coil if the density function is $\delta(x, y, z) = kz$ ($k > 0$).

5) Compute the following:

a. $\int_C 2xy dx + (x^2 + y^2) dy$; $C: x = \cos t, y = \sin t, 0 \leq t \leq \frac{\pi}{2}$

b. $\int_C (2x + y) dx + (x^2 - y) dy$; $C: x = t, y = t^2, 0 \leq t \leq 1$

6) Compute $\int_C ydx + x^2dy$ where C is the path from point $(0,0)$ to point $(2,4)$ given by the equation:

a. $y = 2x$

b. $y = x^2$

7) Compute $\int_{(1,1)}^{(4,2)} (x+y)dx + (y-x)dy$ along each of the following curves:

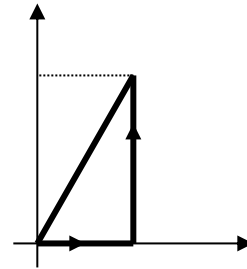
a. The parabola $y^2 = x$

b. A line segment

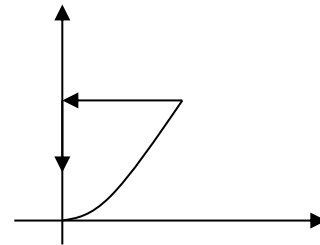
c. The line segments from $(1,1)$ to $(1,2)$ and then to $(4,2)$

d. The curve: $x = 2t^2 + t + 1, y = t^2 + 1 \quad 0 \leq t \leq 1$

8) Compute $\int_C x^2ydx + xdy$ where the path C is as in the figure:



9) Compute $\int_C (x - y^2)dx + dy$ where C is as in the figure:



10) If $\mathbf{F}(x, y, z) = (3x^2 - 6yz)\mathbf{i} + (2y + 3xz)\mathbf{j} + (1 - 4xyz^2)\mathbf{k}$,

compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ from $(0,0,0)$ to $(1,1,1)$ along the paths:

a. $x = t, y = t^2, z = t^3$.

b. The lines from $(0,0,0)$ to $(0,0,1)$, then to $(0,1,1)$ and then to $(1,1,1)$.

c. The line from $(0,0,0)$ to $(1,1,1)$.

11) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where:

a. $F(x, y) = \langle x^2y^3, -y\sqrt{x} \rangle$, $C: r(t) = \langle t^2, -t^3 \rangle$, $0 \leq t \leq 1$

b. $F(x, y, z) = \langle \sin x, \cos y, xz \rangle$, $C: r(t) = \langle t^3, -t^2, t \rangle$, $0 \leq t \leq 1$

12) Answer the following:

a. Compute the work done by the force field $\mathbf{F}(x, y) = x^3y\mathbf{i} + (x - y)\mathbf{j}$ on a particle which moves along the parabola $y = x^2$ from $(-2, 4)$ to $(1, 1)$.

b. How would your answer change if the particle moved from $(1, 1)$ to $(-2, 4)$?

13) Compute the work done by the force field $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ on a particle which moves along the path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ ($0 \leq t \leq 1$).

Remark on Notation

A line integral of type II has different notation in the technical literature:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C (f, g, h) \cdot (dx, dy, dz) = \int_C f dx + g dy + h dz$$
$$\int_C \mathbf{A} \cdot d\mathbf{r} = \int_C (A_1, A_2, A_3) \cdot (dx, dy, dz) = \int_C A_1 dx + A_2 dy + A_3 dz$$

Answer Key

1) a π . b $\frac{16}{3}$. c $\frac{3\sqrt{5}}{2}$. d $\frac{5}{6}(\sqrt{2}+1)$.

2) a $2\sqrt{2}\pi\left(1+\frac{4\pi^2}{3}\right)$. b $\frac{567}{2}$.

3) 6

4) $\sqrt{5}k\pi^2$

5) a $\frac{1}{3}$. b $\frac{4}{3}$.

6) a $\frac{28}{3}$. b $\frac{32}{3}$.

7) a $\frac{34}{3}$. b. 11 c.14 d. $\frac{32}{3}$

8) $\frac{1}{2}$

9) $\frac{4}{5}$

10) a. 2 b. -3 c. $\frac{6}{5}$

11) a. $-\frac{59}{105}$ b. $\frac{6}{5} - \sin 1 - \cos 1$

12) a. 3 b. -3

13) 1