proprep™

Workbook

Markov Chains

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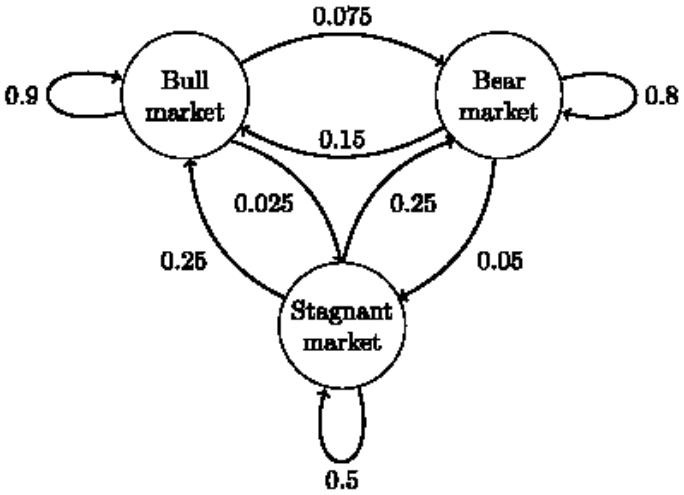
Questions

1) Suppose today is sunny.
Then what's the probability it will be sunny/rainy:

a. Tomorrow?
b. In 2 days?
c. n days from now?
d. After ∞ days?

2) A hypothetical stock market, during a given week, can exhibit one of three trends: a bull market, a bear market, or a stagnant market. The 3 trends are our 3 "states". The probability of a trend the following week depends only on the trend in this week, as in the following transition diagram:



a. Labelling the state space {1 = bull, 2 = bear, 3 = stagnant}, what is the transition matrix?
b. Find the steady state of this system.

Answer Key

1) a. $u_1 = u_0 A = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0.8 & 0.2\\ 0.6 & 0.4\end{bmatrix} = \begin{bmatrix}0.8 & 0.2\end{bmatrix}$

In words: sunny 80%, rainy 20%.

b. $u_2 = u_0 A^2 = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0.76 & 0.24\\ 0.72 & 0.28\end{bmatrix} = \begin{bmatrix}0.76 & 0.24\end{bmatrix}$

c. $u_n = \frac{1}{4}\begin{bmatrix}3+0.2^n & 1-0.2^n\end{bmatrix}$

d. $\begin{bmatrix}0.75 & 0.25\end{bmatrix}$

2) a. Let's call it A, then: $A = \begin{bmatrix}0.9 & 0.075 & 0.025\\ 0.15 & 0.8 & 0.05\\ 0.25 & 0.25 & 0.5\end{bmatrix}$.

b. Bull market 62.5%, bear market 31.25%, stagnant market 6.25%.