

Workbook



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The Laplace Transform

The Laplace Transform

Questions

Compute the following questions:

1) $\mathcal{L}(t^2 + 4t - 2)$

2) $\mathcal{L}\left(\frac{1}{2}t^4 + \frac{2}{\sqrt{\pi}}\sqrt{t} + 1\right)$

3) $\mathcal{L}(e^{-4t} + 10e^{2t})$

4) $\mathcal{L}(\cosh 4t)$

5) $\mathcal{L}(\sinh 10t)$

6) $\mathcal{L}(\sin 2t \cos 2t)$

7) $\mathcal{L}(\sin 2t \cos t)$

8) $\mathcal{L}(\sin^2 t)$

9) $\mathcal{L}(\cos^2 4t)$

10) $\mathcal{L}(t^2 \sin 4t)$

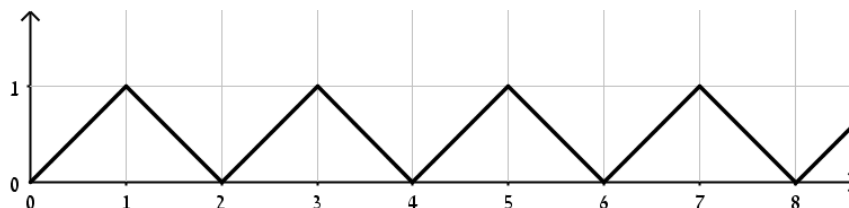
11) $\mathcal{L}(t^4 e^{2t})$

12) $\mathcal{L}(e^{2t} \sin 4t)$

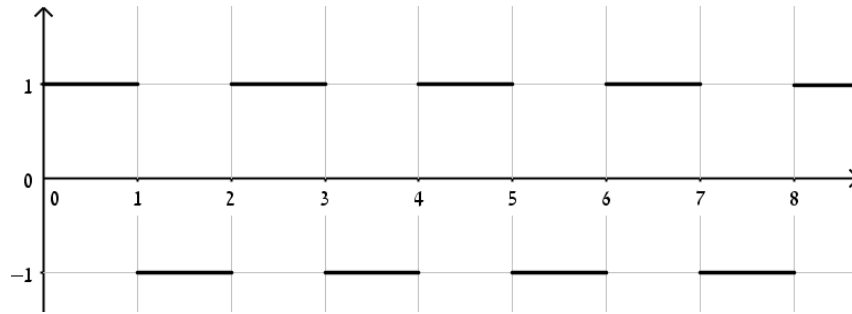
13) $g(t) = \begin{cases} t & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}$

14) $g(t) = \begin{cases} t & 0 < t \leq 1 \\ 2-t & 1 < t \end{cases}$

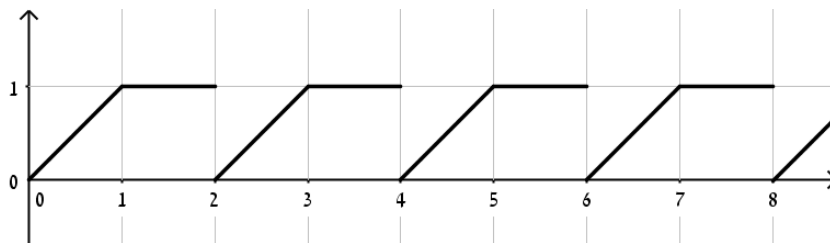
15) Compute the Laplace Transform of the following periodic function:



16) Compute the Laplace Transform of the following periodic function:



17) Compute the Laplace Transform of the following periodic function:



18) Define and sketch the step function $u(t)$.

19) Sketch the function $f(t) = u(t-2) - u(t-3)$, where $u(t)$ is the step function.

20) Express the function $f(t) = \begin{cases} 0 & t < 4 \\ 1 & t \geq 4 \end{cases}$ in terms of the step function.

21) If $f(t)$ is a function and $u(t)$ is the step function, write the Laplace Transforms of

- $u(t)$
- $u(t-k)$
- $u(t-k)f(t-k)$, assuming that we have the transform of $f(t)$.

Compute the Laplace Transform of the function:

22) $g(t) = \begin{cases} 0 & t < 4 \\ (t-4)^2 & t \geq 4 \end{cases}$

23) $g(t) = \begin{cases} 0 & t < 4 \\ t^2 & t \geq 4 \end{cases}$

Answer Key

1) $\frac{2}{s^3} + \frac{4}{s^2} - \frac{2}{s}$

2) $\frac{1}{s+4} + 10\frac{1}{s-2}$

3) $\frac{1}{s+4} + 10\frac{1}{s-2}$

4) $\frac{1}{2}\left[\frac{1}{s-4} + \frac{1}{s+4}\right]$

5) $\frac{1}{2}\left[\frac{1}{s-10} - \frac{1}{s+10}\right]$

6) $\frac{2}{s^2+16}$

7) $\frac{1}{2}\frac{5}{s^2+25} - \frac{1}{2}\frac{1}{s^2+1}$

8) $\frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+4}$

9) $\frac{1}{2}\cdot\frac{1}{s} + \frac{1}{2}\frac{s}{s^2+64}$

10) $(-1)^2\left(\frac{4}{s^2+16}\right)^n = \frac{8(3s^2-16)}{(s^2+16)^3}$

11) $\frac{4}{(s-2)^2+16}$

12) $\frac{4}{(s-2)^2+16}$

13) $\frac{1-e^{-s}}{s^2}$

14) $\frac{1-2e^{-s}}{s^2}$

15) $\frac{1-e^{-s}}{s^2(1+e^{-s})}$

16) $\frac{1-e^{-s}}{s(1+e^{-s})}$

17) $\frac{1-e^{-s}-se^{-2s}}{s^2(1-e^{-2s})}$

18) $u_k(t) = \begin{cases} 0 & t < k \\ 1 & t \geq k \end{cases}$

19) $\begin{cases} 0 & t < 2 \\ 1 & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases}$

20) $f(t) = \begin{cases} 0 & t < 4 \\ 1 & t \geq 4 \end{cases} = u(t-4)$

21) a. $\mathcal{L}(u(t)) = \frac{1}{s}$

b. $\mathcal{L}(u(t-k)) = \frac{e^{-ks}}{s}$

c. $\mathcal{L}(u(t-k)f(t-k)) = e^{-ks}\mathcal{L}(f(t))$

22) $\frac{2e^{-4s}}{s^3}$

23) $\frac{2e^{-4s}(8s^2+4s+1)}{s^3}$

The Inverse Laplace Transform

Questions

Compute the following Inverse Laplace Transforms:

1) $L^{-1}\left(\frac{1}{s}\right)$

2) $L^{-1}\left(\frac{1}{s^4}\right)$

3) $L^{-1}\left(\frac{1}{s-10}\right)$

4) $L^{-1}\left(\frac{1}{s^2+4}\right)$

5) $L^{-1}\left(\frac{s}{s^2+4}\right)$

6) $L^{-1}\left(\frac{1}{(s-10)^2+4}\right)$

7) $L^{-1}\left(\frac{s}{(s-2)^2+4}\right)$

8) $L^{-1}\left(\frac{s}{(s^2+4)^2}\right)$

9) $L^{-1}\left(\frac{1}{(s^2+4)^2}\right)$

10) $L^{-1}\left(\frac{1}{\sqrt{s}}\right)$

11) $L^{-1}\left(\frac{1}{s^2-4}\right)$

12) $L^{-1}\left(\frac{5-s}{s^2+5s}\right)$

13) $L^{-1}\left(\frac{s}{s^2+5s+6}\right)$

14) $L^{-1}\left(\frac{s^2+s-1}{s^3-s}\right)$

15) $L^{-1}\left(\frac{6s^2+4s-6}{s^3-7s-6}\right)$

16) $L^{-1}\left(\frac{10s}{s^4-13s^2+36}\right)$

17) $L^{-1}\left(\frac{8s}{(s-2)^2(s+2)}\right)$

18) $L^{-1}\left(\frac{5-s}{s^3+s^2}\right)$

19) $L^{-1}\left(\frac{9s+36}{s^3+6s^2+9s}\right)$

20) $L^{-1}\left(\frac{1}{(s^2-2s+1)(s^2-4s+4)}\right)$

21) $L^{-1}\left(\frac{1}{s^2 + 2s + 3}\right)$

22) $L^{-1}\left(\frac{1}{s^2 + s + 1}\right)$

23) $L^{-1}\left(\frac{2s^2 + s - 1}{(s^2 + 1)(s - 3)}\right)$

24) $L^{-1}\left(\frac{2s^2 + 2s + 1}{(s^2 + 1)(s + 2)}\right)$

25) $L^{-1}\left(\frac{3}{(s^2 + 1)(s^2 + 4)}\right)$

26) $L^{-1}\left(\frac{1}{s(s^2 + 1)^2}\right)$

27) $L^{-1}\left(\frac{25s^2}{(s - 1)(s^2 + 4)^2}\right)$

28) Answer the following questions:

- Compute the Inverse Laplace Transform: $\mathcal{L}^{-1}\left(\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right)$.
- Express the result as a piecewise function and sketch its graph.

Compute the following Inverse Laplace Transforms:

29) $\mathcal{L}^{-1}\left(\frac{e^{-4s}}{s+1} + \frac{e^{2s}}{s^2+1}\right)$

30) $\mathcal{L}^{-1}\left(\frac{e^{-10s}}{(s-1)(s-2)}\right)$

31) Given the Laplace Transform $F(s) = \frac{e^{-s} + 2}{s}$, of a function $f(t)$.

Compute $f(0)$ and $f(\infty)$.

Answer Key

1) 1

3) e^{10t}

5) $\cos 2t$

7) $e^{2t} \left\{ \cos 2t + 2 \frac{1}{2} \sin 2t \right\}$

9) $\frac{1}{2 \cdot 2^3} (\sin 2t - 2t \cos 2t)$
16

11) $\frac{e^{2t} - e^{-2t}}{4}$

13) $3e^{-3t} - 2e^{-2t}$

15) $e^{-t} + 2e^{-2t} + 3e^{3t}$

17) $e^{2t} + 4te^{2t} - e^{-2t}$

19) $4 - 4e^{-3t} - 3te^{-3t}$

21) $\frac{1}{\sqrt{2}} e^{-t} \sin \sqrt{2}t$

23) $\sin t + 2e^{3t}$

25) $\sin t - \frac{1}{2} \sin 2t$

27) $e^t - \cos 2t - \frac{1}{2} \sin 2t + 5t \sin 2t + \frac{5}{4} (\sin 2t - 2t \cos 2t)$

28) a. $3 - 4u(t-1) \cdot (t-1) + 4u(t-3) \cdot (t-3)$ b. $\begin{cases} 3 & t < 1 \\ 7 - 4t & 1 < t < 3 \\ -5 & t \geq 3 \end{cases}$

29) $u(t-4)e^{-(t-4)} + u(t+2)\sin(t+2)$

30) $u(t-10)(e^{t-10} - e^{2(t-10)})$

31) $f(t) = u(t-1) + 2 = \begin{cases} 2 & t < 1 \\ 3 & t \geq 1 \end{cases}, f(0) = 2, f(\infty) = 3$

2) $t^3 / 3!$

4) $0.5 \sin 2t$

6) $0.5e^{10t} \sin 2t$

8) $L^{-1} \left(\frac{s}{(s^2 + 2^2)^2} \right) = \frac{1}{4} t \sin 2t$

10) $L^{-1} \left(\frac{1}{\sqrt{s}} \right) = \frac{1}{\sqrt{\pi} \sqrt{t}}$

12) $1 - 2e^{-5t}$

14) $1 + \frac{1}{2} e^t - \frac{1}{2} e^{-t}$

16) $2(\cosh 3t - \cosh 2t)$

18) $-6 + 5t + 6e^{-2t}$

20) $2e^t + te^t - 2e^{2t} + te^{2t}$

22) $\frac{1}{\sqrt{\frac{3}{4}}} e^{-\frac{1}{2}t} \sin \sqrt{\frac{3}{4}}t$

24) $\cos t + e^{-2t}$

26) $1 - \cos t - \frac{1}{2} t \sin t$

Solving ODEs with the Laplace Transform

Questions

Solve:

1) $y' + 4y = e^{-3t}$; $y(0) = 0$

2) $y'' + 4y' + 4y = 10e^{-2t}$; $y(0) = -1$, $y'(0) = 4$

3) $y'' - 4y' = 16$; $y(0) = -1$, $y'(0) = -4$

4) $y'' + 4y' = 8t + 2$; $y(0) = y'(0) = 0$

5) $4y'' - 4y' = te^t + e^t$; $y(0) = y'(0) = \frac{1}{4}$

6) $y'' - 3y' + 2y = u(t-4)$; $y(0) = y'(0) = 0$, where $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$ is the unit step function.

7) $y'' + y' = f(t)$; $y(0) = y'(0) = 0$, where $f(t) = \begin{cases} 0 & t < 1 \\ 2 & t \geq 1 \end{cases}$.

8) $y'' + 5y' + 6y = h(t)$; $y(0) = y'(0) = 0$, where $h(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & t \geq 2 \end{cases}$.

9) $y'''' + 4y'' + 5y' + 2y = 10\cos t$; $y(0) = y'(0) = 0$, $y''(0) = 3$

Answer Key

1) $y(t) = e^{-3t} - e^{-4t}$

2) $y(t) = e^{-2t} (5t^2 - 2t - 1)$

3) $y(t) = -4t - 1$

4) $y(t) = t^2$

5) $y(t) = \frac{1}{8}e^t (t^2 + 2)$

6) $y(t) = u(t-4)(0.5 - e^{t-4} + e^{2(t-4)})$

7) $y(t) = 2u(t-1) \cdot (-1 + (t-1) + e^{-(t-1)})$

8) $y(t) = -u(t-2) \frac{1}{6} [1 - 3e^{-2(t-2)} + 2e^{-3(t-2)}] + \frac{1}{6} [1 - 3e^{-2t} + 2e^{-3t}]$

9) $y(t) = -\cos t + 2 \sin t + 2e^{-t} - 2te^{-t} - e^{-2t}$

