proprep™

Workbook

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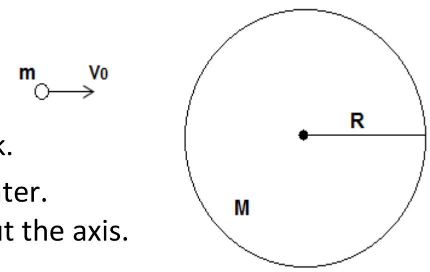
Rigid Body

Angular Momentum of a Rigid Body

Questions

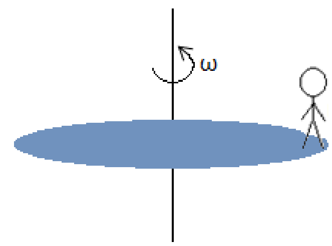
1) Ball and Disk Collision.

A disk of mass M and radius R is at rest and attached to a frictionless axis at its center.
A small ball of mass m moves at a velocity v_0 towards the disk.
The ball hits the disk from the left, a distance d above Its center.
The ball sticks on to the disk, which then begins rotating about the axis.
What is the initial angular velocity of the system?



2) Man Jumps off a Disk.

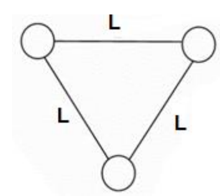
A disk of radius R and mass M is rotating about an axis at its center a constant angular velocity ω_0.
A man of mass m stands at the edge of the disk.
The man jumps off the disk. His velocity at this moment is v_0, in the radial direction, relative to the ground.
What is the disk's angular velocity after the jump?



3) Three Balls.

Three identical balls of mass m are placed at the corners of an equilateral triangle. The balls are conjoined by three massless rods of length L (the sides of the triangle).

a. Find the systems centre of mass.



It is now given that the system moves with angular velocity ω about the centre of mass. At some moment, when the system is in the position described by the diagram, the lower ball disconnects from the system.

b. Find the velocity of the disconnected ball after the break off.
c. Find the velocity of the remaining system's centre of mass.
d. Find the angular velocity of the remaining system about its centre of mass.

Rotational Energy of a Rigid Body

Questions

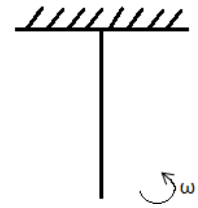
4) Rotating Rod.

A rod, of length L and mass M, is attached to the ceiling.
The rod rotates at an initial angular velocity of ω_0.
What is the maximum angle which the rod will reach?

5) Ball Hits Rod.

A ball of mass m hits a rod, which is attached to the ceiling, a distance x from the rod's axis of rotation.
The rod is of length L and mass M.

a. What is the angular velocity of the system right after collision?
b. What is the maximum angle the rod will reach?
c. Find which length of x will cause the force being applied by the ceiling to the rod to be equal to 0.



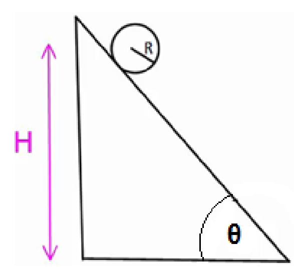
Analysis Through Forces And Moments, Rolling Without Slipping

Questions

6) Ball on a Slope.

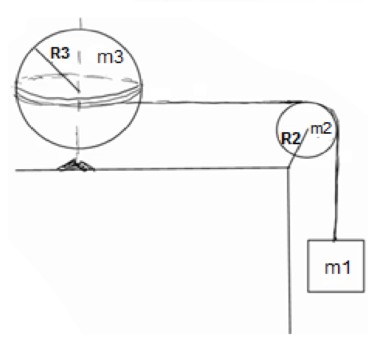
A ball of radius R is placed at a height H on a slope of angle $\theta°$.
The ball begins to roll down the slope without slipping.

a. What is the velocity of the ball at the bottom of the slope?
b. What is the ball's acceleration?



7) Ball and Pulley.

A ball is nailed to a table.
It rotates around the axis perpendicular to the table.
A rope is wound around the center of the ball, it rests on a non-ideal pulley system. A mass m_1, is attached to its end. m_2 and R_2 are the mass and radius of the pulley, and m_3 and R_3 are the mass and radius of the ball.
The system begins at rest.
Find each body's acceleration, as well as the tension in the rope.

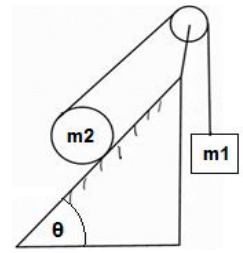


8) Yoyo and a Mass.

A yo-yo (a ball with a string wound around it) of mass m_2 and radius R is placed on a slope of angle $\theta°$.

The yo-yo's string is attached, via an ideal pulley, to mass m_1.

We are told that the yo-yo rolls, without slipping, down the slope and that there is friction between the yo-yo and the slope.



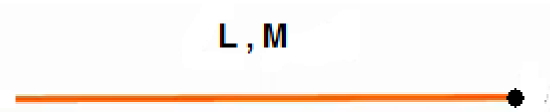
a. In which direction is the static friction? Find the movement of the system.
b. Find the accelerations of the bodies and the size of the friction.

9) Falling Horizontal Rod.

A rod of mass M (with uniform density) and length L is hung from one end to a wall such that it is free to rotate about the point of attachment.

The rod is released from a horizontal position.

a. Find the angular acceleration and the acceleration of the rod's cener of mass at the moment of release.
b. Find the force that the axis (connecting the rod to the wall) exerts at the moment of release.



The rod falls until it is perpendicular to the ground.

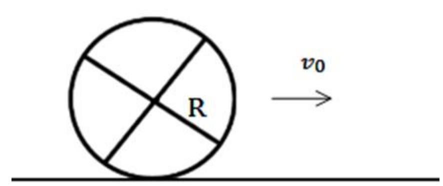
c. Find the angular acceleration of the rod at this moment (when the rod is perpendicular to the ground).
d. Repeat sections a. and b., but this time the rod is perpendicular to the ground.

Rolling with Slipping

10) Ball Slipping without Rotation.

A homogenous ball of mass M and initial velocity without rotating (no angular velocity).

Find its final velocity if it is known that the coefficient of friction is kinetic.

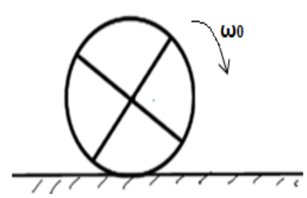


11) Spinning Ball.

A homogenous ball of mass M is held in the air, where it rotates about its center of mass with an angular velocity ω_0.

The ball is lowerd to the ground whilst rotating.

What is the ball's final velocity, if the coefficient of friction is μ_k?



End of Chapter Question

12) Falling Pencil.

A pencil stands perpendicular to the table.
The pencil begins to fall to the right.
When the angle between the pencil and the axis perpendicular to the table reaches θ_1 the pencil begins to slip.

a. For all angles θ such that $\theta < \theta_1$:
   i. What is the angular velocity of the pencil?
   ii. What is the angular acceleration of the pencil?
   iii. Find the acceleration vector of the pencil's center of mass.
   iv. Find the size and direction of the frictional force.
   v. Find the normal force.

b. Find the static coefficient of friction, μ_s.

Answer Key

1) $\omega=\dfrac{mv_0 d}{R^2\left(\frac{1}{2}M+m\right)}$

2) $\omega=\dfrac{\left(\frac{1}{2}M+m\right)\omega_0}{\frac{1}{2}M\omega}$

3) a. $y_{cm}=\dfrac{\sqrt{3}L}{6}=\dfrac{L}{2\sqrt{3}}$ b. $\vec{v}_3=-\dfrac{\omega L}{\sqrt{3}}\hat{x}$ c. $\vec{v}_{cm}=\dfrac{\omega L}{2\sqrt{3}}\hat{x}$ d. $\omega'=\dfrac{2\omega}{L}\left(\dfrac{2L}{3}-\dfrac{1}{6}\right)$

4) $\cos\theta=1-\dfrac{L\omega_0^2}{3g}$

5) a. $\omega=\dfrac{mu_0 x}{mx^2+\frac{1}{3}ML^2}$ b. $\cos\theta=-\dfrac{\frac{1}{2}I_T\omega^2}{g\left(\frac{ML}{2}+mx\right)}+1$ c. $x=\dfrac{\frac{mu_0}{\omega}-\frac{ML}{2}}{m}$

6) a. $v_{cm}=\sqrt{\dfrac{10}{7}gh}$ b. $a_x=\dfrac{5}{3}g\sin\theta$

7) Solution in the recording.

8) Solution in the recording.

9) a. $\alpha=\dfrac{3}{2}\dfrac{g}{L},\quad a_{cm}=\dfrac{3g}{4}\hat{y},\quad a_x=0$ b. $F_x=0,\quad F_y=\dfrac{1}{4}mg$ c. $\omega=\sqrt{\dfrac{3g}{L}}$

d. $\alpha=0,\quad a_{cm}=-\dfrac{3g}{2};\quad F_x=0,\quad F_y=-\dfrac{1}{2}Mg$

10) $v_f=\dfrac{5}{7}v_0$

11) $v\left(\dfrac{2\omega_0 R}{7\mu_k g}\right)=\dfrac{2}{7}\omega_0 R$

12) a.(i) $\omega=\sqrt{3\dfrac{g}{L}(1-\cos\theta)}$ (ii) $\alpha=\dfrac{3g}{2L}\sin\theta$ (iii) $\vec{a}=-3\dfrac{g}{2}(1-\cos\theta)\hat{r}+\dfrac{3g}{4}\sin\theta\hat{\theta}$

(iv) $f_s=\dfrac{3}{2}g\left(\dfrac{1}{2}\sin\theta\cos\theta+\cos\theta-1\right)$ (v) $N=mg\left(1-\dfrac{3}{2}(1-\cos\theta)\cos\theta-\dfrac{3}{4}\sin^2\theta\right)$

b. $\mu_R=\dfrac{3\left(\frac{1}{2}\sin\theta_1\cos\theta_1+\cos\theta_1-1\right)}{2m\left(1-\frac{3}{2}\cos\theta_1+\frac{3}{2}\cos^2\theta_1-\frac{3}{4}\sin^2\theta_1\right)}$