proprep

Workbook

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Applicatios of Fourier Series on Differential Equations

Sturm-Liouville Problems

Questions

1) $\begin{cases} y''+\lambda y=0 \ , \ 0\le x\le 1 \\ y'(0)=0 \\ y'(1)=0 \end{cases}$

2) $\begin{cases} y''+\lambda y=0 \ , \ 0\le x\le 1 \\ y(0)=0 \\ y(1)+y'(1)=0 \end{cases}$

3) $\begin{cases} y''+\lambda y=0 \ , \ 0\le x\le 1 \\ y(0)+y'(0)=0 \\ y(1)=0 \end{cases}$

4) $\begin{cases} y''+\lambda y=0 \ , \ 0\le x\le l \\ y(0)=0 \\ y'(l)=0 \end{cases}$

5) $\begin{cases} y''+\lambda y=0 \ , \ 0\le x\le \pi \\ y'(0)=0 \\ y(\pi)=0 \end{cases}$

6) $\begin{cases} y''-2y'+(1+\lambda)y=0 \ , \ 0\le x\le 1 \\ y(0)=0 \\ y(1)=0 \end{cases}$

7) Find eigenvalues and eigenfunctions for the following Sturm-Liouville problem:

$$\begin{cases} y''+\lambda y=0 \ , \ \ 0\le x\le L \\ y(0)=0 \\ y(L)=0 \end{cases}$$

8) Find eigenvalues and eigenfunctions for the following Sturm-Liouville problem:

$$\begin{cases} y''+\lambda y=0 \ , \ \ 0\le x\le L \\ y'(0)=y'(L)=0 \end{cases}$$

Answer Key

To view the answers to those exercises, please refer to the appropriate videos on site.

The Heat Equation

Questions

1) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_t = u_{xx} & 0 \le x \le \pi, \quad t \ge 0 \\ u(x,0) = \sin x \\ u(0,t) = u(\pi,t) = 0 \end{cases}$$

2) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_t = u_{xx} & 0 \le x \le \pi, \quad t \ge 0 \\ u(x,0) = \dfrac{1}{2} + 3\sin^2 x \\ u_x(0,t) = u_x(\pi,t) = 0 \end{cases}$$

3) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_t = 16u_{xx} & 0 \le x \le 3, \quad t \ge 0 \\ u(x,0) = x \\ u_x(0,t) = u_x(3,t) = 0 \end{cases}$$

4) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_t = 9u_{xx} & (0 \le x \le 2, \quad t \ge 0) \\ u(x,0) = 6 + 4\cos\left(\dfrac{3\pi}{2}x\right) \\ u_x(0,t) = u_x(2,t) = 0 \end{cases}$$

5) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_t = 9u_{xx} & (0 \le x \le 2, \quad t \ge 0) \\ u(x,0) = x \\ u_x(0,t) = u_x(2,t) = 0 \end{cases}$$

Answer Key

1) $u(x,t) = e^{-t}\sin x$

2) $u(x,t) = 2 - \frac{3}{2}e^{-4t}\cos(2x)$

3) $u(x,t) = \frac{3}{2} + \sum_{k=1}^{\infty} \frac{-12}{\pi^2(2k-1)^2} e^{-\left(\frac{4\pi(2k-1)}{3}\right)^2 t} \cos\left(\frac{\pi(2k-1)}{3}x\right)$

4) $u(x,t) = 6 + 4e^{-\frac{81\pi^2}{4}t}\cos\left(\frac{3\pi}{2}x\right)$

5) $u(x,t) = 1 + \sum_{k=1}^{\infty} \frac{-8}{\pi^2(2k-1)^2} e^{-\left(\frac{3\pi(2k-1)}{2}\right)^2 t} \cos\left(\frac{\pi(2k-1)}{2}x\right)$

The Wave Equation

Questions

1) Solve the following Wave Equation using separation of variables.

$$\begin{cases} u_{tt} = u_{xx} & 0 \le x \le 1, \quad t \ge 0 \\ \left.\begin{array}{l} u(x,0) = \frac{1}{2}\sin(\pi x) - 7\sin(5\pi x) \\ u_t(x,0) = 0 \end{array}\right\} & \text{initial conditions} \\ u(0,t) = u(1,t) = 0 & \text{boundary conditions} \end{cases}$$

2) Solve the following Wave Equation using separation of variables.

$$\begin{cases} u_{tt} = u_{xx} & 0 \le x \le 1, \quad t \ge 0 \\ \left.\begin{array}{l} u(x,0) = 1 \\ u_t(x,0) = 0 \end{array}\right\} & \text{initial conditions} \\ u(0,t) = u(1,t) = 0 & \text{boundary conditions} \end{cases}$$

3) Solve the following Heat Equation using separation of variables.

$$\begin{cases} u_{tt} = 4u_{xx} & 0 \le x \le 1, \quad t \ge 0 \\ \left.\begin{array}{l} u(x,0) = -2x - 1 \\ u_t(x,0) = 0 \end{array}\right\} & \text{initial conditions} \\ u(0,t) = u(1,t) = 0 & \text{boundary conditions} \end{cases}$$

Answer Key

1) $u(x,t)=\frac{1}{2}\sin(\pi x)\cos(\pi t)-7\sin(5\pi x)\cos(5\pi t)$

2) $u(x,t)=\sum_{k=1}^{\infty}\frac{-4}{\pi(2k-1)}\sin((2k-1)\pi x)\cos((2k-1)\pi t)$

3) $u(x,t)=\sum_{n=1}^{\infty}2\frac{3(-1)^n-1}{\pi n}\sin(n\pi x)\cos(2n\pi t)$