

Workbook



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Fourier Transform

Introduction to Fourier Transform

Questions

- 1) Compute the Fourier transform of $\chi_{[-1,1]}(x) = \begin{cases} 1 & x \in [-1,1] \\ 0 & \text{else} \end{cases}$ (rectangular window function).
- 2) Compute the Fourier transform of $f(x) = \begin{cases} 1-|x| & |x| < 1 \\ 0 & \text{else} \end{cases}$.
- 3) Compute the Fourier transform of $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{else} \end{cases}$.
- 4) Compute the Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 2 & 1 < |x| < 2 \\ 0 & \text{else} \end{cases}$.
- 5) Compute the Fourier transform of $f(x) = \begin{cases} e^{-ax} & x > 0 \\ e^{bx} & x \leq 0 \end{cases}$ where $a, b > 0$.
- 6) Compute the Fourier transform of $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{else} \end{cases}$.
- 7) Compute the Fourier transform of $f(x) = \begin{cases} 1 & 0 \leq x < 1 \\ 2 & 1 \leq x < 2 \\ 0 & \text{else} \end{cases}$.
- 8) Compute the Fourier transform of $f(x) = \begin{cases} e^x & 0 \leq x < 1 \\ 0 & \text{else} \end{cases}$.

9) Compute the Fourier transform of $f(x) = \begin{cases} e^{2ix} & -1 < x < 1 \\ 0 & \text{else} \end{cases}$.

10) Compute the Fourier transform of $f(x) = \begin{cases} \sin x & -1 < x < 1 \\ 0 & \text{else} \end{cases}$.

11) Compute the Fourier transform of $f(x) = \begin{cases} x & |x| < a \\ 0 & \text{else} \end{cases} \quad (a > 0)$.

12) Prove that there's no function $f(x)$ in $G(\mathbb{R})$ such that $\hat{f}(\omega) = \begin{cases} 1 - |\omega| & |\omega| \leq \frac{1}{2} \\ 0 & \text{else} \end{cases}$.

Answer Key

1) $\frac{\sin \omega}{\pi \omega}$

2) $\frac{1 - \cos \omega}{\pi \omega^2}$

3) $\frac{1}{2\pi(1+i\omega)}$

4) $\frac{2 \sin 2\omega - \sin \omega}{\pi \omega}$

5) $\frac{1}{2\pi} \left\{ \frac{1}{a+i\omega} + \frac{1}{b-i\omega} \right\}$

6) $\frac{\sin \omega + i(\cos \omega - 1)}{2\pi \omega}$

7) $\frac{1 + e^{-i\omega} - 2e^{-2i\omega}}{2\pi i \omega}$

8) $\frac{1}{2\pi} \frac{e^{1-i\omega} - 1}{1-i\omega}$

9) $\frac{1}{\pi} \frac{\sin(2-\omega)}{2-\omega}$

10) $-\frac{i}{2\pi} \left[\frac{\sin(1-\omega)}{1-\omega} - \frac{\sin(1+\omega)}{1+\omega} \right]$

11) $-\frac{i}{\pi} \frac{\sin(\omega a) - \omega a \cos(\omega a)}{\omega^2}$

12) By the properties of the Fourier transform,

\hat{f} has to be continuous for all ω

but the function above isn't continuous

for $|\omega| = \frac{1}{2}$.

