proprep

# Workbook

# Table of Contents

# The Derivative of a Function

## Basic Derivatives of Functions

### Questions

**1)** Find the first derivative:

a. $f(x)=4$
b. $g(x)=\frac{e+\sqrt{2}}{2}$
c. $h(x)=x^4$
d. $y=\frac{1}{x^2}$
e. $f(x)=\sqrt{x}$
f. $y=\frac{1}{\sqrt{x}}$
g. $y=4x^{10}+\frac{1}{x}$
h. $y=x+x^2$
i. $y=4x^2+8x^3-5$
j. $y(x)=\frac{2}{x}-ex$
k. $y=\sqrt{2}x^2+2ex$
l. $y(t)=\frac{4}{t}+\sqrt[3]{t}$

**2)** Find the first derivative:

a. $y=\left(x^2+3\right)\left(x-1\right)$
b. $y=\left(4x+10\right)\left(\sqrt{x}-1\right)$
c. $y=\left(x-1\right)\left(x-1\right)\left(x-2\right)$
d. $y=\frac{4x+10}{x^2-x}$
e. $y=\frac{x^2+4x-1}{2x-3}$
f. $y=\frac{ex+1}{ex-1}$
g. $y=\left(4x+10\right)^3$
h. $y=\left(x^2+1\right)^5$
i. $y=\sqrt{\left(x^2+x+1\right)^3}$
j. $y=\left(2x+1\right)^3\left(4x-5\right)^4$
k. $y=\frac{\left(2x+3\right)^4}{\left(x-5\right)^3}$
l. $y=\frac{1}{\sqrt[3]{4x+1}}$

## Answer Key

**1)** a. $f'(x)=0$ b. $g'(x)=0$ c. $h'(x)=4x$

d. $y'(x)=\frac{-2}{x^3}$ e. $f'(x)=\frac{1}{2\sqrt{x}}$ f. $z'(x)=-\frac{1}{2x^{1.5}}$

g. $y'(x)=40x^9-\frac{1}{x^2}$ h. $y'(x)=1+2x$ i. $y'(x)=8x+24x^2$

j. $y'(x)=-\frac{2}{x^2}-e$ k. $y'(x)=2\sqrt{2}x+2e$ l. $y'(t)=-\frac{4}{t^2}+\frac{1}{3\sqrt[3]{t^2}}$

**2)** a. $y'(x)=3x^2-2x+3$ b. $y'(x)=6\sqrt{x}-4+\frac{5\sqrt{x}}{x}$

c. $y'(x)=3x^2-8x+5$ d. $y'(x)=\frac{2(2x^2+10x-5)}{x^2(x-1)^2}$

e. $y'(x)=\frac{2(x^2-35-5)}{(3-2x)^2}$ f. $y'(x)=\frac{-2e}{(ex-1)^2}$

g. $y'(x)=12(4x+10)^2$ h. $y'(x)=10x(x^2+1)^4$

i. $y'(x)=\left(3x+\frac{3}{2}\right)\sqrt{x^2+x+1}$ j. $y'(x)=14(2x+1)^2(4x-5)^3(2x-1)$

k. $y'(x)=\frac{(2x+3)^3(2x-49)}{(x-5)^4}$ l. $y'(x)=-\frac{4}{3(4x+1)^{\frac{4}{3}}}$

## Derivative of Exponents and Logarithmic Functions

### Questions

**1)** Find the first derivative:

a. $y = e^x$

b. $y = 4e^x + 2x^3$

c. $y = e^x\left(x^2 + x + 4\right)$

d. $y = \frac{e^x}{x^2 - x}$

e. $y = e^{4x-1} + e^{2x}$

f. $y = e^{-x}\left(x + 1\right)$

g. $y = \frac{e^{2x} - x}{e^{4+x}}$

h. $y = e^{\sqrt{x}}$

i. $y = \frac{e^{\pi x}}{x - 2}$

j. $y = \frac{1}{\sqrt{e^{4x} + 1}}$

k. $y = \sqrt[3]{e^{x^2+1} + 1}$

l. $y = \frac{e^{-x^2}}{x}$

**2)** Find the first derivative and second derivatives:

a. $y = \frac{\ln x}{x}$

b. $y = \ln^2 x + \frac{1}{\ln x}$

c. $y = e^{2x}\ln\left(x^2 + 4\right)$

d. $y = \frac{\ln x}{e^x}$

## Answer Key

**1)**

a. $y'(x)=e^x$

b. $y'(x)=4e^x+6x^2$

c. $y'(x)=e^x\left(x^2+3x+5\right)$

d. $y'(x)=\dfrac{e^x\left(x^2-3x+1\right)}{\left(x^2-x\right)^2}$

e. $y'(x)=4e^{4x-1}+2e^{2x}$

f. $y'(x)=-xe^{-x}$

g. $y'(x)=e^{-x-4}\left(x+e^{2x}-1\right)$

h. $y'(x)=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

i. $y'(x)=\dfrac{e^{\pi x}\left(\pi x-(2\pi+1)\right)}{(x-2)^2}$

j. $y'(x)=\dfrac{-2e^{4x}}{\left(e^{4x}+1\right)\sqrt{e^{4x}+1}}$

k. $y'(x)=\dfrac{2e^{x^2}x}{3\sqrt[3]{\left(e^{x^2}+1\right)^2}}$

l. $y'(x)=\dfrac{e^{-x^2}\left(2x^2+1\right)}{x^2}$

**2)**

a. $y'(x)=\dfrac{1-\ln(x)}{x^2}$ , $y''(x)=\dfrac{2\ln(x)-3}{x^3}$

b. $y'(x)=\dfrac{2\ln^2(x)-1}{x\ln^2(x)}$ , $y''(x)=\dfrac{2\ln^4(x)+2\ln^3(x)+\ln(x)+2}{x^2\ln^3(x)}$

c. $y'(x)=2e^{2x}\left(\ln\left(x^2+4\right)+\dfrac{x}{x^2+4}\right)$ , $y''(x)=\dfrac{2e^{2x}\left(4x^3-x^2+2\left(x^2+4\right)^2\ln\left(x^2+4\right)+16x+4\right)}{\left(x^2+4\right)^2}$

d. $y'(x)=\dfrac{\left(\frac{1}{x}-\ln(x)\right)}{e^x}$ , $y''(x)=\dfrac{e^x\left(-x^2\ln(x)-2x-1\right)}{x^2}$

# Trigonometric Derivatives

## Questions

**1)** Find the first derivative:

a. $f(x)=\sin x$

b. $g(x)=\sin 4x$

c. $y=\cos(0.5x)$

d. $y=\sin^2 x$

e. $f(x)=\cos^4(5x)$

f. $z(x)=\sqrt{\sin 2x}$

g. $y=\sin x\cos 3x$

h. $y=\dfrac{\sin x-1}{\cos 2x+2}$

i. $y=x^3\sin 4x$

j. $y(x)=\ln(\cos(x))$

k. $y=e^{\sin 2x}\ln x$

l. $y(t)=\sin(\cos(x))$

## Answer key

**1)**

a. $f'(x)=\cos(x)$

b. $g'(x)=4\cos(4x)$

c. $y'(x)=-0.5\sin(0.5x)$

d. $y'(x)=2\sin(x)\cos(x)$

e. $f'(x)=-500\sin^3(x)$

f. $f'(x)=\dfrac{\cos(2x)}{\sqrt{\sin(2x)}}$

g. $y'(x)=\cos(x)\cos(3x)-\sin(x)\sin(3x)-3\sin(x)$

h. $y'(x)=\dfrac{\cos(x)\cos(2x)+2\cos(x)+2\sin(x)\sin(2x)-2\sin(2x)}{(\cos 2x+2)^2}$

i. $y'(x)=3x^2\sin(4x)+4x^3\cos(4x)$

j. $y'(x)=-\tan(x)$

k. $y'(x)=e^{\sin 2x}\left(2\cos(2x)\ln(x)+\dfrac{1}{x}\right)$

l. $y'(t)=-\sin(t)\cdot\cos(\cos(t))$

# Derivative of Power Functions

## Questions

**1)** Find the first derivative:

a. $y = x^{2x}$

b. $y = x^{\ln x}$

c. $y = (\ln x)^x$

d. $y = (x^2 + 1)^{4x}$

e. $y = x^{x^2+1}$

f. $y = (\sqrt{x})^{\sqrt{2x}}$

g. $y = x^{e^x}$

h. $y = \left(x^{x^x}\right)$

i. $y = (\sin x)^x$

j. $y = x^{\cos 2x}$

k. $y = (\tan x)^{2x}$

l. $y = (\sin x)^{\ln x}$

## Answer key

**1)** a. $y'(x) = 2x^{2x}(\ln(x)+1)$

b. $y'(x) = \dfrac{2e^{\ln^2(x)}\ln(x)}{x}$

c. $y'(x) = \ln(x)^x \cdot \left(\ln(\ln(x)) + \dfrac{1}{x}\right)$

d. $y'(x) = 4(x^2+1)^{4x} \cdot \left(\ln(x^2+1) + \dfrac{2x^2}{x^2+1}\right)$

e. $y'(x) = x^{x^2+1}\left(2x\ln(x) + \dfrac{x^2+1}{x}\right)$

f. $y'(x) = \sqrt{x}^{\sqrt{2x}} \cdot \dfrac{1}{2}\left(\dfrac{\ln(x)}{\sqrt{2x}} + \dfrac{\sqrt{2x}}{x}\right)$

g. $y'(x) = x^{e^x} \cdot^x \left(\ln(x) + \dfrac{1}{x}\right)$

h. $y'(x) = x^{x^x+x}\left(\ln^2(x) + \ln(x) + x^{x-1}\right)$

i. $y'(x) = (\sin(x))^x \left[\ln(\sin(x)) + x\cot(x)\right]$

j. $y'(x) = x^{\cos(2x)}\left[-\sin(2x)2\ln(x) + \dfrac{1}{x}\cos(2x)\right]$

k. $y'(x) = (\tan(x))^{2x}\left[2\left(\ln(\tan(x)) + \dfrac{x\sin(x)}{\cos^2 x}\right)\right]$

l. $y'(x) = \sin(x)^{\ln(x)}\left[\dfrac{\ln(\sin(x))}{x} + \ln(x)\cot(x)\right]$

# Implicit Differentiation

## Questions

**1)** Find $y'$:

a. $x^2+y^2=1$ b. $x^2y^3=x+y^2$ c. $\frac{y^2+x}{y^3-4x}=1$

d. $\sqrt{y}+\sqrt{x}=1$ e. $(y+2)^3=xy$ f. $e^x+e^y=1$

g. $\ln x+\ln y=y$ h. $(\ln y)^2+y\ln x=1$ i. $\sin y+\cos x=y^2$

j. $x\tan y=\sqrt{y}$ k. $x^y+y^x=1$ l. $y^{\ln x}+x^{\ln y}=4$

**2)** Find the first and the second derivatives:

a. $x^2+y^3=1$ b. $x^2y^3=x+y$

c. $\ln x+\ln y=1$ d. $\sin x+\sin y=x$

## Answer Key

**1)**

a. $y'(x)=-\frac{x}{y}$

b. $y'(x)=\frac{1-2xy^3}{3xy^2-2y}$

c. $y'(x)=\frac{-y^3-4y^2}{-y^4-8xy-3xy^2}$

d. $y'(x)=-\sqrt{\frac{y}{x}}$

e. $y'(x)=\frac{y}{3(y+2)^2-x}$

f. $y'(x)=-e^{x-y}$

g. $y'(x)=\frac{y}{x(y-1)}$

h. $y'(x)=-\frac{y^2}{z(2\ln y+y\ln x)}$

i. $y'(x)=\frac{\sin x}{\cos y-2y}$

j. $y'(x)=\frac{\tan y}{\frac{1}{3\sqrt[3]{y^2}}-\frac{x}{\cos^2 y}}$

k. $y'(x)=-\frac{y^x(\ln y+1)}{x^y\left(\ln x+\frac{x}{y}\right)}$

l. $y'(x)=-\frac{y\ln y}{x\ln x}$

**2)**

a. $y'(x)=\frac{-2x}{3y^2}$ , $y''(x)=\frac{2y^2+8x^2}{9y^6}$

b. $y'(x)=\frac{1-2xy^3}{3x^2y^2-1}$ , $y''(x)=-2\left[y^3+3xy^2\frac{1-2xy^3}{3x^2y^2-1}\right]\left[3x^2y^2-1\right]-6\left(1-2xy^3\right)\left(xy^2+2x^2y\frac{1-2xy^2}{3x^2y^2-1}\right)$

c. $y'(x)=-\frac{y}{x}$ , $y''(x)=\frac{2y}{x^2}$

d. $y'(x)=\frac{1-\cos(x)}{\cos(y)}$ , $y''(x)=\frac{(1+\sin(x))\cos(y)}{\sin(y)(\cos(x)-1)}$

# Calculations Using the Definition of Derivative

## Questions

**1)** Find the first derivative of the given function using the definition of the derivative:

a. $f(x)=x^2$

b. $g(x)=x^2+4x+1$

c. $f(x)=x^3$

d. $y=\frac{1}{x}$

e. $f(x)=\sqrt{x}$

f. $z(x)=\ln x$

g. $y=e^x$

h. $y=\sin 2x$

## Answer Key

**1)** a. $y'(x)=2x$

b. $y'(x)=2x+4$

c. $y'(x)=3x^2$

d. $y'(x)=\frac{-1}{x^2}$

e. $y'(x)=\frac{1}{2\sqrt{x}}$

f. $y'(x)=\frac{1}{x}$

g. $y'(x)=e^x$

h. $y'(x)=2\cos(x)$

# The Derivative of an Inverse of a Function

## Questions

**1)** Prove that $(\ln x)' = \frac{1}{x}$. Use the rule of derivative of the inverse function.

**2)** Prove that $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$. Use the rule of derivative of the inverse function.

**3)** Prove that $(\arcsin x)' = \frac{1}{\sqrt{1+x^2}}$. Use the rule of derivative of the inverse function.

**4)** Prove that $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$. Use the rule of derivative of the inverse function.

**5)** Supposed that $f^{-1}$ is the inverse function of a differentiable function $f$ and that: $f(2)=7$ , $f'(2)=\sqrt{7}$. Find $(f^{-1})=(7)$.

## Answer Key

**1-4)** To view the answers to the exercises, please refer to the appropriate videos on site.

**5)** $\frac{1}{\sqrt{7}}$

## Logarithmic Differentiation

### Questions

**1)** Given the followinf function: $y=\sqrt[4]{\frac{10x-1}{x+1}}\cdot\sqrt[10]{(2x+1)^7}$ , find $y'$.

**2)** Given the followinf function: $y=\left(\sqrt[4]{10x+1}\right)^{2x}$ , find $y'$.

**3)** Find the equation of the line that is tanfent to the curve: $f(x)=x^3-4x^2+2x-5$
At the point on the curve where $x=1$.
Does the line intersect the curve at any other point?

### Answer Key

**1)** $y'=y\left(\frac{1}{4}\frac{10}{10x-1}-\frac{1}{4}\times\frac{1}{x+1}+\frac{7}{10}\frac{2}{2x+1}\right)$

**2)** $y'=\frac{1}{4}2^x y\left[\ln 2\ln(10x+1)+\frac{10}{10x+1}\right]$

**3)** $y=-3x-3$, no.