

Workbook



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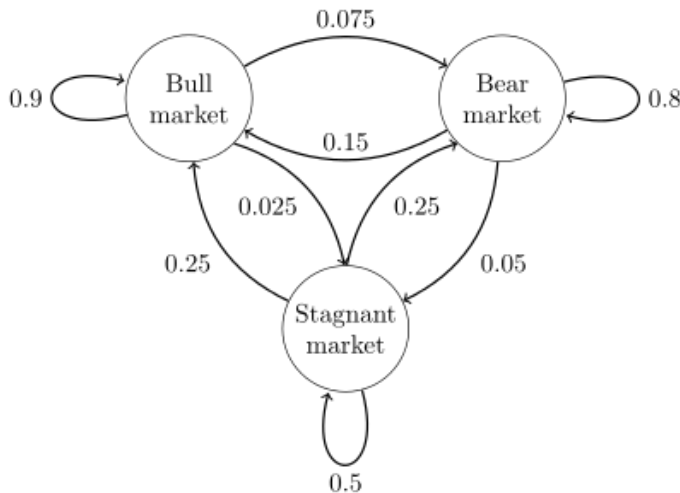
Markov Chains

Markov Chains

Questions

- 1) Suppose today is sunny. Then what's the probability it will be sunny/rainy:
 - a. Tomorrow?
 - b. In 2 days?
 - c. n days from now?
 - d. After ∞ days?

- 2) A hypothetical stock market, during a given week, can exhibit one of three trends: a bull market, a bear market, or a stagnant market. The 3 trends are our 3 "states". The probability of a trend the following week depends only on the trend in this week, as in the following transition diagram:



- a. Labelling the state space $\{1 = \text{bull}, 2 = \text{bear}, 3 = \text{stagnant}\}$, what is the transition matrix?
- b. Find the steady state of this system.

Answer Key

1) a. $u_1 = u_0 A = [1 \ 0] \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} = [0.8 \ 0.2]$ - In words: sunny 80%, rainy 20%.

b. $u_2 = u_0 A^2 = [1 \ 0] \begin{bmatrix} 0.76 & 0.24 \\ 0.72 & 0.28 \end{bmatrix} = [0.76 \ 0.24]$

c. $u_n = \frac{1}{4} [3 + 0.2^n \quad 1 - 0.2^n]$

d. $[0.75 \ 0.25]$

2) a. Let's call it A , then: $A = \begin{bmatrix} 0.9 & 0.075 & 0.025 \\ 0.15 & 0.8 & 0.05 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$.

b. Bull market 62.5%, bear market 31.25%, stagnant market 6.25%.