proprep

Workbook

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Graphical and Numerical Methods

Graphical Methods

Questions

1) Given the differential equation $y' = x + y$.

a. Sketch line elements for the points (x, y), where $x, y \in \{-2, -1, 0, 1, 2\}$.

b. Compare with a computer-aided sketch of the direction field.

2) Given the differential equation $y' = x + y$.

a. Describe the isoclines associated with this DE and sketch a few (say, 3).

b. Draw a few line elements on each of these.

c. Compare with a computer-aided sketch of the direction field.

3) Given the differential equation $y' = x + y$.

a. Show a computer-aided sketch of the associated direction field.

b. Use this to manually sketch the solutions to the DE passing through the points $(0,0)$, $(0,1)$ and $(0,-1)$.

4) Given the differential equation $y' = x^2 + y^2 - 2$.

a. Describe the isoclines associated with this DE and sketch those for slopes $c = -1, 0, 2$.

b. Draw a few line elements on each of these.

c. Compare with a computer-aided sketch of the direction field.

d. Use the DF to manually sketch the solutions to the DE passing through the points $(0,0)$, $(0,2)$ and $(0,-2)$.

Answer Key

To View the answers to the exercises, please refer to the appropriate videos on site.

Numerical Methods

Questions

1) Given the IVP $y' = x + y$; $y(0) = 0$.

a. Estimate $y(1)$ using Euler's method with step-size $h = 0.25 = \frac{1}{4}$.
[Manual computation]

b. Estimate $y(1)$ using Euler's method with step-size $h = 0.05 = \frac{1}{20}$.
[Computer-aided]

c. As above but with $h = 0.01 = \frac{1}{100}$.

d. Verify that $y = e^x - x - 1$ is the solution to the IVP and compute $y(1)$.

e. Round the results to 3 decimal places.
What can we say here about step-size and accuracy?

2) Consider the following IVP: $\begin{cases} y' = 2x(1+y) \\ y(0) = 0 \end{cases}$.

a. Find the first 3 Picard approximations, i.e. $y_n(x)$ for $n = 1, 2, 3$.

b. Find a general expression for the n^{th} approximation $y_n(x)$ [no need to prove].

c. Find the limit $y(x) = \lim_{n\to\infty} y_n(x)$ and show that this is a solution of the IVP.

Answer Key

To View the answers to the exercises, please refer to the appropriate videos on site.