proprep

Workbook

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N^th Order Linear Equations

Linear, Homogeneous, Constant Coefficients

Questions

Solve the following equations:

1) $y'''-2y''-3y'=0$

2) $y^{(4)}+3y'''-15y''-19y'+30y=0$

3) $y'''-2y''-y'+2y=0$

4) $y^{(4)}-5y''+4y=0$

5) $y^{(4)}-y=0$

6) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+20y=0$

7) $y^{IV}+y=0 \quad [y^{IV}=y^{(4)}=y'''']$

8) $y^{(6)}-y''=0$

9) $\left(D^5+3D^4+2D^3-2D^2-3D-1\right)y=0$

10) $y^{(8)}+8y^{(4)}+16y=0$

11) $z'''-6z''+12z'-8z=0$

12) $y^{(4)}-4y=0$

13) $x^{(6)}-3x^{(4)}+3x''-x=0 \quad , \quad x=x(t)$

14) $\begin{cases} y'''-y''+y'-y=0 \\ y(0)=3, y'(0)=4, y''(0)=-1 \end{cases}$

15) $\begin{cases} y''''-3y'''+6y''-12y'+8y=0 \\ y(0)=2, y'(0)=5, y''(0)=-19, y'''(0)=-47 \end{cases}$

16) Given a homogeneous ODE of the 6^th order with constant coefficients, one of whose solutions is $x^2e^x\cos 2x$.

a. Find the general solution of the equation.

b. Find the equation.

Answer Key

1) $y = c_1 + c_2 e^{-x} + c_3 e^{3x}$

2) $y = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x} + c_4 e^{-5x}$

3) $y = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-x}$

4) $y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{-2x}$

5) $y = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

6) $y = c_1 e^{-4x} + e^{x}[c_2 \cos 2x + c_3 \sin 2x]$

7) $y = e^{\frac{\sqrt{2}}{2}x}\left(c_1 \cos \frac{\sqrt{2}}{2}x + c_2 \sin \frac{\sqrt{2}}{2}x\right) + e^{-\frac{\sqrt{2}}{2}x}\left(c_3 \cos \frac{\sqrt{2}}{2}x + c_4 \sin \frac{\sqrt{2}}{2}x\right)$

8) $y = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-x} + \cos x + \sin x$

9) $y = c_1 e^{x} + c_2 e^{-x} + c_3 x e^{-x} + c_4 x^2 e^{-x} + c_5 x^3 e^{-x}$

10) $+ e^{-x}[c_5 \cos x + c_6 \sin x] + x e^{-x}[c_7 \cos x + c_8 \sin x]$

11) $y = c_1 e^{2x} + c_2 x e^{2x} + c_3 x^2 e^{2x}$

12) $y = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} + c_3 \cos \sqrt{2}x + c_4 \sin \sqrt{2}x$

13) $y = c_1 e^{t} + c_2 t e^{t} + c_3 t^2 e^{t} + c_4 e^{-t} + c_5 t e^{-t} + c_6 t^2 e^{-t}$

14) $y = e^{x} + 2\cos x + 3\sin x$

15) $y = e^{x} - 2e^{2x} + 3\cos 2x + 4\sin 2x$

16) a. $y = e^{x}[c_1 \cos 2x + c_2 \sin 2x] + x e^{x}[c_3 \cos 2x + c_4 \sin 2x] + x^2 e^{x}[c_5 \cos 2x + c_6 \sin 2x]$

b. $y^{(6)} - 6y^{(5)} + 27y^{(4)} - 68y''' + 135y'' - 150y' + 125y = 0$

Method of Undetermined Coefficients

Questions

Solve the following equations:

1) $y''' - 2y'' - 3y' = 2\sin x - 4\cos x$

2) $y^{(4)} + 3y''' - 15y'' - 19y' + 30y = -28e^{2x}$

3) $y''' - 2y'' - y' + 2y = 2x^3 - 3x^2 - 12x + 14$

4) $y''' - 3y' + 2y = e^x$

5) $y''' - y'' + y' - y = \sin x$

Answer Key

1) $y = c_1 + c_2 e^{-x} + c_3 e^{3x} + \sin x$

2) $y = c_1 e^x + c_2 e^{-2x} + c_3 e^{3x} + c_4 e^{-5x} + e^{2x}$

3) $y = c_1 e^{2x} + c_2 e^x + c_3 e^{-x} + x^3 + 4$

4) $y = c_1 e^x + c_2 x e^x + c_3 e^{-2x} + \frac{1}{6} x^2 e^x$

5) $y = c_1 e^x + c_2 \cos x + c_3 \sin x + \frac{1}{4} x(\cos x - \sin x)$

The Wronskian and its Uses

Questions

Solve the following equations:

1) Is it possible that $y_1(x)=x,\ y_2(x)=x^2,\ y_3(x)=x^3$ are three solutions of a linear homogeneous 3rd order ODE $y'''+p(x)y''+q(x)y'+r(x)y=0$, with continuous coefficients $p(x),\ q(x),\ r(x)$ on the interval $[0,\pi]$?

2) We're given the functions $y_1(x)=4-x$, $y_2(x)=4+x$, $y_3(x)=20+x$.

a. Compute the Wronskian of the functions.

b. Determine whether or not the functions are l.i. on $(-\infty,\infty)$.

c. Answer b. again, after noting that the three functions are solutions of $y''=0$.

3) Answer the following Questions

a. Let $y_1(x),\ y_2(x),\ y_3(x)$ be functions which are thrice continuously differentiable on an interval I , and such that their Wronskian is nonzero on I .
Prove that there exists an ODE $y'''+p(x)y''+q(x)y'+r(x)y=0$ with continuous coefficients on I , such that $y_1(x),\ y_2(x),\ y_3(x)$ are three of its solutions.

b. Find an equation $y'''+p(x)y''+q(x)y'+r(x)y=0$ with continuous coefficients on $x>0$ such that $y_1(x)=x,\ y_2(x)=x^2,\ y_3(x)=x^3$ are three of its solutions.

Answer Key

1) $y_1(x)=x,\ y_2(x)=x^2,\ y_3(x)=x^3$, can't possibly be solutions of the ODE as described.

2) a. $W=0$ b. Thus, the functions are linearly dependent.

c. When the functions are solutions of this kind of ODE the problem becomes easier. We can immediately conclude from $W=0$ that they are linearly dependent.

3) a. $y'''-y''\frac{|\square|}{W}+y'\frac{|\square|}{W}-y\frac{|\triangle|}{W}=0$

b. $y'''-\frac{3}{x}y''+\frac{6}{x^2}y'-\frac{6}{x^3}y=0 \ , \ (x>0)$