proprep

Workbook

Table of Contents

The Laplace Transform

The Laplace Transform

Questions

Compute the following questions:

1) $\mathcal{L}\left(t^2+4t-2\right)$

2) $\mathcal{L}\left(\frac{1}{2}t^4+\frac{2}{\sqrt{\pi}}\sqrt{t}+1\right)$

3) $\mathcal{L}\left(e^{-4t}+10e^{2t}\right)$

4) $\mathcal{L}\left(\cosh 4t\right)$

5) $\mathcal{L}\left(\sinh 10t\right)$

6) $\mathcal{L}\left(\sin 2t\cos 2t\right)$

7) $\mathcal{L}\left(\sin 2t\cos t\right)$

8) $\mathcal{L}\left(\sin^2 t\right)$

9) $\mathcal{L}\left(\cos^2 4t\right)$

10) $\mathcal{L}\left(t^2\sin 4t\right)$

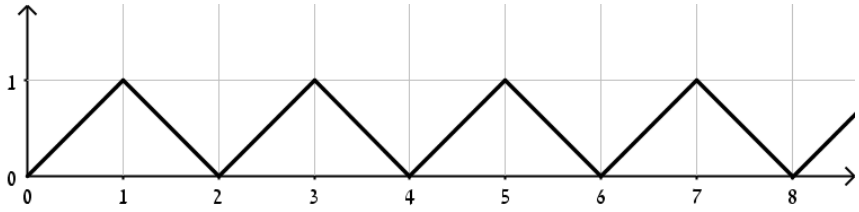
11) $\mathcal{L}\left(t^4e^{2t}\right)$

12) $\mathcal{L}\left(e^{2t}\sin 4t\right)$

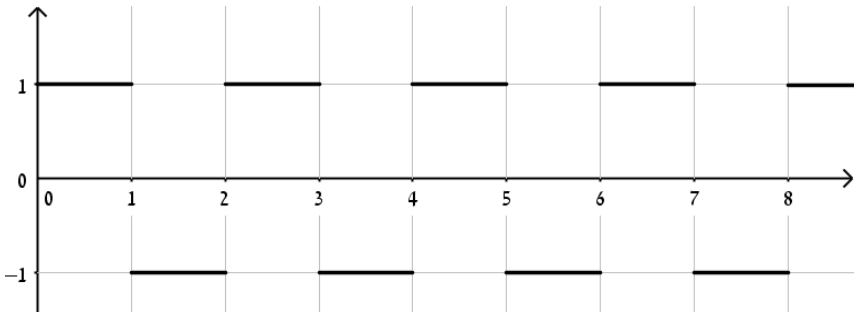
13) $g(t)=\begin{cases}t & 0<t\le 1\\ 1 & t>1\end{cases}$

14) $g(t)=\begin{cases}t & 0<t\le 1\\ 2-t & 1<t\end{cases}$

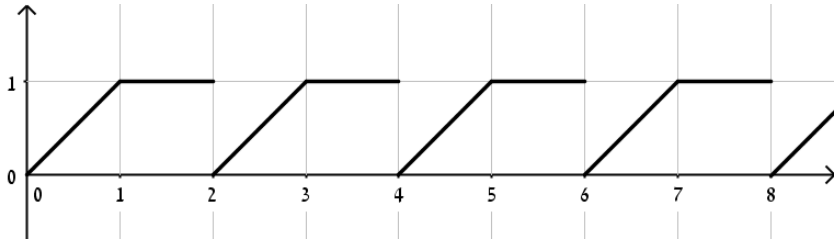
15) Compute the Laplace Transform of the following periodic function:



16) Compute the Laplace Transform of the following periodic function:



17) Compute the Laplace Transform of the following periodic function:



18) Define and sketch the step function $u(t)$.

19) Sketch the function $f(t)=u(t-2)-u(t-3)$,
where $u(t)$ is the step function.

20) Express the function $f(t)=\begin{cases}0 & t<4\\ 1 & t\geq 4\end{cases}$ in terms of the step function.

21) If $f(t)$ is a function and $u(t)$ is the step function, write the Laplace Transforms of

a. $u(t)$

b. $u(t-k)$

c. $u(t-k)\,f(t-k)$, assuming that we have the transform of $f(t)$.

Compute the Laplace Transform of the function:

22) $g(t)=\begin{cases}0 & t<4\\ (t-4)^2 & t\geq 4\end{cases}$

23) $g(t)=\begin{cases}0 & t<4\\ t^2 & t\geq 4\end{cases}$

Answer Key

1) $\frac{2}{s^3}+\frac{4}{s^2}-\frac{2}{s}$

2) $\frac{1}{s+4}+10\frac{1}{s-2}$

3) $\frac{1}{s+4}+10\frac{1}{s-2}$

4) $\frac{1}{2}\left[\frac{1}{s-4}+\frac{1}{s+4}\right]$

5) $\frac{1}{2}\left[\frac{1}{s-10}-\frac{1}{s+10}\right]$

6) $\frac{2}{s^2+16}$

7) $\frac{1}{2}\frac{5}{s^2+25}-\frac{1}{2}\frac{1}{s^2+1}$

8) $\frac{1}{2}\cdot\frac{1}{s}-\frac{1}{2}\frac{s}{s^2+4}$

9) $\frac{1}{2}\cdot\frac{1}{s}+\frac{1}{2}\frac{s}{s^2+64}$

10) $(-1)^2\left(\frac{4}{s^2+16}\right)''=\frac{8\left(3s^2-16\right)}{\left(s^2+16\right)^3}$

11) $\frac{4}{(s-2)^2+16}$

12) $\frac{4}{(s-2)^2+16}$

13) $\frac{1-e^{-s}}{s^2}$

14) $\frac{1-2e^{-s}}{s^2}$

15) $\frac{1-e^{-s}}{s^2\left(1+e^{-s}\right)}$

16) $\frac{1-e^{-s}}{s\left(1+e^{-s}\right)}$

17) $\frac{1-e^{-s}-se^{-2s}}{s^2\left(1-e^{-2s}\right)}$

18) $u_k(t)=\begin{cases}0 & t<k\\ 1 & t\geq k\end{cases}$

19) $\begin{cases}0 & t<2\\ 1 & 2\leq t<3\\ 0 & t\geq 3\end{cases}$

20) $f(t)=\begin{cases}0 & t<4\\ 1 & t\geq 4\end{cases}=u(t-4)$

21) a. $\mathcal{L}\left(u(t)\right)=\frac{1}{s}$ b. $\mathcal{L}\left(u(t-k)\right)=\frac{e^{-ks}}{s}$ c. $\mathcal{L}\left(u(t-k)\,f(t-k)\right)=e^{-ks}\mathcal{L}\left(f(t)\right)$

22) $\frac{2e^{-4s}}{s^3}$

23) $\frac{2e^{-4s}\left(8s^2+4s+1\right)}{s^3}$

The Inverse Laplace Transform

Questions

Compute the following Inverse Laplace Transforms:

1) $L^{-1}\left(\frac{1}{s}\right)$

2) $L^{-1}\left(\frac{1}{s^4}\right)$

3) $L^{-1}\left(\frac{1}{s-10}\right)$

4) $L^{-1}\left(\frac{1}{s^2+4}\right)$

5) $L^{-1}\left(\frac{s}{s^2+4}\right)$

6) $L^{-1}\left(\frac{1}{(s-10)^2+4}\right)$

7) $L^{-1}\left(\frac{s}{(s-2)^2+4}\right)$

8) $L^{-1}\left(\frac{s}{\left(s^2+4\right)^2}\right)$

9) $L^{-1}\left(\frac{1}{(s^2+4)^2}\right)$

10) $L^{-1}\left(\frac{1}{\sqrt{s}}\right)$

11) $L^{-1}\left(\frac{1}{s^2-4}\right)$

12) $L^{-1}\left(\frac{5-s}{s^2+5s}\right)$

13) $L^{-1}\left(\frac{s}{s^2+5s+6}\right)$

14) $L^{-1}\left(\frac{s^2+s-1}{s^3-s}\right)$

15) $L^{-1}\left(\frac{6s^2+4s-6}{s^3-7s-6}\right)$

16) $L^{-1}\left(\frac{10s}{s^4-13s^2+36}\right)$

17) $L^{-1}\left(\frac{8s}{(s-2)^2(s+2)}\right)$

18) $L^{-1}\left(\frac{5-s}{s^3+s^2}\right)$

19) $L^{-1}\left(\frac{9s+36}{s^3+6s^2+9s}\right)$

20) $L^{-1}\left(\frac{1}{\left(s^2-2s+1\right)\left(s^2-4s+4\right)}\right)$

21) $L^{-1}\left(\dfrac{1}{s^2+2s+3}\right)$

22) $L^{-1}\left(\dfrac{1}{s^2+s+1}\right)$

23) $L^{-1}\left(\dfrac{2s^2+s-1}{(s^2+1)(s-3)}\right)$

24) $L^{-1}\left(\dfrac{2s^2+2s+1}{(s^2+1)(s+2)}\right)$

25) $L^{-1}\left(\dfrac{3}{(s^2+1)(s^2+4)}\right)$

26) $L^{-1}\left(\dfrac{1}{s(s^2+1)^2}\right)$

27) $L^{-1}\left(\dfrac{25s^2}{(s-1)(s^2+4)^2}\right)$

28) Answer the following questions:

a. Compute the Inverse Laplace Transform: $\mathcal{L}^{-1}\left(\dfrac{3}{s}-\dfrac{4e^{-s}}{s^2}+\dfrac{4e^{-3s}}{s^2}\right)$.

b. Express the result as a piecewise function and sketch its graph.

Compute the following Inverse Laplace Transforms:

29) $\mathcal{L}^{-1}\left(\dfrac{e^{-4s}}{s+1}+\dfrac{e^{2s}}{s^2+1}\right)$

30) $\mathcal{L}^{-1}\left(\dfrac{e^{-10s}}{(s-1)(s-2)}\right)$

31) Given the Laplace Transform $F(s)=\dfrac{e^{-s}+2}{s}$, of a function $f(t)$.

Compute $f(0)$ and $f(\infty)$.

Answer Key

1) 1

2) $t^3/3!$

3) e^{10t}

4) $0.5\sin 2t$

5) $\cos 2t$

6) $0.5e^{10t}\sin 2t$

7) $e^{2t}\left\{\cos 2t + 2\frac{1}{2}\sin 2t\right\}$

8) $L^{-1}\left(\frac{s}{\left(s^2+2^2\right)^2}\right)=\frac{1}{4}t\sin 2t$

9) $\frac{1}{2\cdot 2^3}\left(\sin 2t - 2t\cos 2t\right)$

16

10) $L^{-1}\left(\frac{1}{\sqrt{s}}\right)=\frac{1}{\sqrt{\pi}\sqrt{t}}$

11) $\frac{e^{2t}-e^{-2t}}{4}$

12) $1-2e^{-5t}$

13) $3e^{-3t}-2e^{-2t}$

14) $1+\frac{1}{2}e^{t}-\frac{1}{2}e^{-t}$

15) $e^{-t}+2e^{-2t}+3e^{3t}$

16) $2(\cosh 3t-\cosh 2t)$

17) $e^{2t}+4te^{2t}-e^{-2t}$

18) $-6+5t+6e^{-2t}$

19) $4-4e^{-3t}-3te^{-3t}$

20) $2e^{t}+te^{t}-2e^{2t}+te^{2t}$

21) $\frac{1}{\sqrt{2}}e^{-t}\sin\sqrt{2}t$

22) $\frac{1}{\sqrt{\frac{3}{4}}}e^{-\frac{1}{2}t}\sin\sqrt{\frac{3}{4}}t$

23) $\sin t+2e^{3t}$

24) $\cos t+e^{-2t}$

25) $\sin t-\frac{1}{2}\sin 2t$

26) $1-\cos t-\frac{1}{2}t\sin t$

27) $e^{t}-\cos 2t-\frac{1}{2}\sin 2t+5t\sin 2t+\frac{5}{4}\left(\sin 2t-2t\cos 2t\right)$

28) a. $3-4u(t-1)\cdot(t-1)+4u(t-3)\cdot(t-3)$

b. $\begin{cases}3 & t<1\\ 7-4t & 1<t<3\\ -5 & t\geq 3\end{cases}$

29) $u(t-4)e^{-(t-4)}+u(t+2)\sin(t+2)$

30) $u(t-10)\left(e^{t-10}-e^{2(t-10)}\right)$

31) $f(t)=u(t-1)+2=\begin{cases}2 & t<1\\ 3 & t\geq 1\end{cases}$, $f(0)=2$, $f(\infty)=3$

Solving ODEs with the Laplace Transform

Questions

Solve:

1) $y'+4y=e^{-3t}\ ;\ y(0)=0$

2) $y''+4y'+4y=10e^{-2t}\ ;\ y(0)=-1\ ,\ y'(0)=4$

3) $y''-4y'=16\ ;\ y(0)=-1\ ,\ y'(0)=-4$

4) $y''+4y'=8t+2\ ;\ y(0)=y'(0)=0$

5) $4y''-4y'=te^{t}+e^{t}\ \ ;\ \ y(0)=y'(0)=\frac{1}{4}$

6) $y''-3y'+2y=u(t-4)\ ;\ y(0)=y'(0)=0$, where $u(t)=\begin{cases}0 & t<0\\ 1 & t\geq 0\end{cases}$ is the unit step function.

7) $y''+y'=f(t)\ ;\ y(0)=y'(0)=0$, where $f(t)=\begin{cases}0 & t<1\\ 2 & t\geq 1\end{cases}$.

8) $y''+5y'+6y=h(t)\ ;\ y(0)=y'(0)=0$, where $h(t)=\begin{cases}1 & 0<t<2\\ 0 & t\geq 2\end{cases}$.

9) $y'''+4y''+5y'+2y=10\cos t\ ;\ y(0)=y'(0)=0, y''(0)=3$

Answer Key

1) $y(t) = e^{-3t} - e^{-4t}$

2) $y(t) = e^{-2t}\left(5t^2 - 2t - 1\right)$

3) $y(t) = -4t - 1$

4) $y(t) = t^2$

5) $y(t) = \frac{1}{8}e^t\left(t^2 + 2\right)$

6) $y(t) = u(t-4)\left(0.5 - e^{t-4} + e^{2(t-4)}\right)$

7) $y(t) = 2u(t-1)\cdot\left(-1 + (t-1) + e^{-(t-1)}\right)$

8) $y(t) = -u(t-2)\frac{1}{6}\left[1 - 3e^{-2(t-2)} + 2e^{-3(t-2)}\right] + \frac{1}{6}\left[1 - 3e^{-2t} + 2e^{-3t}\right]$

9) $y(t) = -\cos t + 2\sin t + 2e^{-t} - 2te^{-t} - e^{-2t}$