proprep

Workbook

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Basic Problems in Probability

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Questions

1) A two-letter word is formed from the letters E, F, and G – not necessarily a real word.

a. List all the possible words.

b. List all the cases for the events:

i. A – The word contains the letter E.

ii. B – All the letters in the word are different.

c. List the cases for event $\overline{A}$.

2) A Pair of Dice is thrown.

a. List the sample space of the trial.
Is the sample space symmetric?

b. List all the possibilities for the following events:

i. The sum of the results is 7.

ii. The product of the results is 12.

c. Calculate the chances of the events defined in b above.

3) A number from 0 to 9 is randomly selected.

a. What is the probability that the number is larger than 5?

b. What is the probability that the number is at most 3?

c. What is the probability that the number is odd?

4) The following table shows the distribution of families according to the number of TV sets that they own. A family is randomly selected.

a. What is the probability that the family does not own a TV set?

b. What is the probability that the family own at least 1 TV set?

c. What is the probability that the family own at least 3 TV sets?

Number of TV Sets	Number of Families
0	22
1	28
2	18
3	22
4	10

5) The following table shows the distribution of families by the number of cars that they own. A family is randomly selected.

a. What is the probability that the family does not own a car?
b. What is the probability that the family own at least 3 cars?
c. What is the probability that the family own than 3 cars?

Number of Cars	Number of Families
0	20
1	40
2	100
3	30
4	10

6) A fair coin (having heads on one side and tails on the other) is tossed three times.

a. List the sample space of the trial.
   Is the sample space symmetrical?
b. List all the possibilities for the following events:
   i. A – Heads was thrown exactly once.
   ii. D – Tails was thrown at least once.
c. What is the complementary event of D?
d. Calculate the chances of the events defined in sections b and c.

Answer Key

1) a. $\Omega = \{EE,\ EF,\ EG\ ,\ FE,\ FF,\ FG,\ GE,\ GF,\ GG\}$

b. $A = \{EE,\ EF,\ EG,\ FE,\ GE\}\ ,\ B = \{EF,\ EG,\ FE,\ FG,\ GE,\ GF\}$

c. $\bar{A} = \{FF,\ FG,\ GF,\ GG\}$

2) The chances of A are $\frac{1}{6}$, and the chances of B are $\frac{1}{9}$.

3) a. 0.4 b. 0.4 c. 0.5

4) a. 0.22 b. 0.78 c. 0.32

5) a. 0.1 b. 0.2 c. 0.8

6) a. $\Omega = \{HHH,\ HHT,\ HTH,\ HTT,\ THH,\ THT,\ TTH,\ TTT\}$, symmetric.

b. $A = \{HTT,\ THT,\ TTH\}$
$D = \{HHT,\ HTH,\ HTT,\ THH,\ THT,\ TTH,\ TTT\}$

c. $\bar{D} = \{HHH\}$

d. $\mathrm{P}(A) = \frac{3}{8}\quad \mathrm{P}(D) = \frac{7}{8}\quad \mathrm{P}(\bar{D}) = \frac{1}{8}$

Operations between Events

Questions

1) A two-letter word is formed from the letters E, F, and G.
The word does not necessarily have any meaning.
We define the following events:
A – The letter E is in the word.
B – The two letters in the word are different.

a. List all the possible results in the intersection of A with B.
b. List all the possible results in the union of A with B.

2) A student takes two semester exams: one in Economics and one in Statistics.
We define the following events:
A – Passing the statistics exam.
B – Passing the economics exam.
Using only the intersection of events, union of events, and complementary events, define the following events, and mark the appropriate area on the diagram.

a. The student passed only the economics exam.
b. The student passed only the statistics exam.
c. The student passed both exams.
d. The student passed at least one exam.
e. The student failed both exams.
f. The student failed the economics exam.

3) You are asked to randomly select a digit (from 0-9).
We define:
A - The selection of an even number.
B - The selection of a number less than 5.

a. List all the results of the following events: A, B, $\overline{B}$, $A \cap B$, $A \cup B$.
b. Calculate the probabilities of all the events in the preceding section.

4) Let Ω denote the sample space and $\varnothing$ denote the empty set.
Let A be an event in the sample space.
The solutions to the following defined events are either Ω, $\varnothing$ *or* A.
Find the solution for each event:

a. $\overline{\overline{A}}$ b. $A \cap \varnothing$ c. $A \cup \varnothing$ d. $A \cap \Omega$

e. $A \cup \Omega$ f. $A \cap \overline{A}$ g. $\overline{\varnothing}$ h. $A \cup \overline{A}$

5) A Persons's Height.
The following events are defined:
A – A person's height is greater than 1.7 meters.
B – A person's height is less than 1.8 meters.
Calculate the following events:

a. $A \cap B$ b. $A \cup B$ c. $\overline{A} \cap B$

d. $\overline{A} \cup \overline{B}$ e. $\overline{\overline{A}}$

6) Three Languages.
We define the following events:
A – A person speaks Chinese.
B – A person speaks French.
C – A person speaks English.
Write the following events using the intersection, union and complementary operators:

a. A person speaks all three languages.
b. A person speaks only Chinese.
c. A person speaks at least one language.
d. A person does not speak English.
e. A person speaks exactly two languages.

7) Two political parties are running in the election – the "Blues" and the "Greys".
The probability of the "Blue" Party passing the voting threshold is 0.08.
The probability of the "Grey" Party passing the voting threshold is 0.20.
The chances that neither will pass the threshold is 76%.

a. What is the probability of at least one of the parties passing the threshold?
b. What is the probability of both parties passing the threshold?
c. What is the probability of only the "Grey" party passing the threshold?

8) Employees and Graduates.
40% of all the employees at work are men, 20% are university graduates, and 10% are female university graduates.

a. What proportion of the employees are male university graduates?
b. What proportion of employees are either males or university graduates?
c. What proportion of the workers are female non-university graduates?

9) The chances of share A rising on a given day are 0.5, and the chances of share B rising on a given day are 0.4. The chances of at least one share rising on a given day are 0.7. Calculate the following probabilities for these two shares on a given day:

a. Both shares rise.
b. Neither share rises.
c. Only share A rises.

10) A pair of dice, one red and one black, is thrown.
We define the following events:
A – The red die is 4 and the black die is 2.
B – The sum of the two dice is 6.
C – The product of the two dice is 10.

a. Are A and B mutually exclusive events?
b. Is event A a subset of event B?
c. Are A and C mutually exclusive events?
d. Are A and C complementary events?

11) The following probabilities are known for events A and B:

$$P(A)=0.6, \qquad P(B)=0.3, \qquad P(\overline{A}\cap\overline{B})=0.1$$

a. Are A and B mutually exclusive events?
b. Calculate: $P(\overline{A}\cap B)$.

12) A coin is tossed twice. We define the following events:
A – The first toss is heads.
B – At least one of the two tosses is heads.
Which of the following statements is correct?

a. A and B are mutually exclusive events.
b. A and B are complementary events.
c. A is a subset of B.
d. B is a subset of A.

13) A lottery is held with 100 tickets being sold. Three of the tickets are for a vacation, two are for computers, and the rest are empty. A person randomly buys one ticket.

a. What are the chances of winning a vacation or a computer?
Are these mutually exclusive events?
b. What is the probability of <u>not</u> winning a prize?

14) Consider the following givens: $P(A)=0.3$, $P(B)=0.25$, $P(A\cup B)=0.49$.

a. Calculate: $P(A\cap B)$.

b. Are A and B mutually exclusive events?

c. What is the probability of only A or only B occurring?

15) A and B are mutually exclusive events.

Assume that: $2P(B\cap\overline{A})=P(A\cap\overline{B})=P(\overline{A}\cap\overline{B})$.

What are the chances of event A and what is the probability of event B?

16) Determine which of the following assertions are correct:

a. $A\cap B=B\cap A$

b. $\overline{A\cup B}=A\cap B$

c. $A\cap B\cap C=A\cap B\cap(C\cup B)$

d. $\overline{A\cap B\cap C}=\overline{A}\cup\overline{B}\cup\overline{C}$

17) Let A and B be events in a sample space.

Let $P(A)=0.3$ and $P(B)=0.2$.

a. Is it possible that $P(A\cup B)=0.4$?

b. Is it possible that $P(A\cup B)=0.6$?

c. If A and B are mutually exclusive events, what is $P(A\cup B)$?

d. If B is a subset of A, what is $P(A\cup B)$?

18) Bank Accounts.

30% of the people in a community have accounts at Bank A, 28% have accounts at Bank B, and 15% have accounts at Bank C. 6% of the people have accounts at both Bank A and Bank B, 5% have accounts at both Bank A and Bank C, and 4% have accounts at both Bank B and Bank C. 1% of the people have accounts at all three banks.

a. What proportion of the adults have an account only at Bank B?

b. What is the probability of a given person having accounts at Bank A and Bank B, but not at Bank C.

c. What is the probability of a given person having accounts at Bank A or Bank C, but not at Bank B?

d. What proportion of people have accounts at only one bank?

e. What proportion of people have accounts at only two banks?

f. What is the probability of an adult having an account at none of these three banks?

g. What proportion of people have an account in at least one of the banks?

19) A company published the following information about people over 21:
40% of the people have a Visa card, 52% have a Master card, 20% have an American-Express card, 15% have both a Visa card and a Master card, 8% have both a Master-card and an American Express card, and 7% have both a Visa card and an American Express card. 13% have none of these three cards.

a. What proportion of people have all three cards?
b. What proportion of people have both a Master card and a Visa card, but not an American Express card?
c. What proportion of people have only one of the three cards?

20) Prove: $P\left(\overline{A}\cap\overline{B}\right)=1-P\left(A\right)+P\left(A\cap B\right)$.

21) A and B are events in the sample space.
Is it correct to say that the chances of exactly one event occurring are $P(A)+P(B)-2P(A\cap B)$?

Answer Key

1) a. $A\cap B=\{EF,\ EG,\ FE,\ GF\}$ b. $A\cup B=\{EE,\ EF,\ EG,\ FE,\ GE,\ FG,\ GF\}$

2) a. $B\cap\bar{A}$ b. $A\cap\bar{B}$ c. $A\cap B$ d. $A\cup B$ e. $\bar{A}\cap\bar{B}=\bar{A}\cup B$ f. $\bar{B}$

3) a. $A=\{0,\ 2,\ 4,\ 6,\ 8\}$, $B=\{1,\ 2,\ 3,\ 4\}$, $\bar{B}=\{5,\ 6,\ 7,\ 8,\ 9\}$, $A\cap B=\{0,\ 2,\ 4\}$, $A\cup B=\{0,\ 2,\ 4,\ 6,\ 8,\ 1,\ 3\}$

b. $P(A)=0.5$, $P(B)=0.5$, $P(\bar{B})=0.5$, $P(A\cap B)=0.3$, $P(A\cup B)=0.7$

4)

a. $\bar{\bar{A}}=A$ b. $A\cap\varnothing=\varnothing$ c. $A\cup\varnothing=A$ d. $A\cap\Omega=A$

e. $A\cup\Omega=\Omega$ f. $A\cap\bar{A}=\varnothing$ g. $\bar{\varnothing}=\Omega$ h. $A\cup\bar{A}=\Omega$

5)

a. $A\cap B=1.7<H<1.8$ b. $A\cup B=$ All possible heights. c. $\bar{A}\cap B=\bar{A}=H\le 1.7$

d. $\bar{A}\cup\bar{B}=\ H\le 1.7\ and\ H\ge 1.8$ e. $\bar{\bar{A}}=A=H>1.7$

6)

a. $A\cap B\cap C$ b. $\bar{A}\cap\bar{B}\cap\bar{C}$ c. $A\cup B\cup C$

d. $\bar{C}$ e. $\left(A\cap B\cap\bar{C}\right)\cup\left(A\cap C\cap\bar{B}\right)\cup\left(B\cap C\cap\bar{A}\right)$

7) a. 0.24 b. 0.04 c. 0.16

8) a. 10% b. 50% c. 50%

9) a. 0.2 b. 0.3 c. 0.3

10) a. No. b. Yes. c. Yes. d. No.

11) a. Yes b. 0.3

12) c

13) a. 0.05; Yes. b. 0.95

14) a. 0.06 b. Not mutually exclusive. c. 0.43

15) $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{5}$

16) a. True . b. False. c. False. d. True.

17) a. Yes. b. No. c. 0.5 d. 0.3

18) a. 0.19 b. 0.05 c. 0.31 d. 0.46

e.0.12 f. 0.41 g. 0.59

19) a. 0.05; b. 0.1; c. 0.67

20) $P(\bar{A}\cap\bar{B})=1-P(A\cup B)\ =1-\left[P(A)+P(B)-P(A\cap B)\right]=1-P(A)-P(B)+P(A\cap B)$

21) Yes.