

# Workbook



## Table of Contents

Confidence Intervals for Sample Means.....	2
Known Population Variance.....	2
Unknown Population Variance .....	5

# Confidence Intervals for Sample Means

## Known Population Variance

---

### Questions

- 1) A researcher is estimating the average salary in Wisconsin. Based on a sample, she determines with a 95% level of certainty that the average weekly salary is between \$1,000 and \$1,400 .
  - a. What is the length of the confidence interval?
  - b. What are the chances that the sample error is greater than \$300?
  
- 2) The average output of a factory is 4,950 products per day over a random set of 100 days. Assume that the population standard deviation is known to be 150 products per day. Estimate the average daily output of a certain factory at a 95% confidence level.
  
- 3) We want to estimate a battery's average lifespan. Assume that the battery's lifespan follows a normal probability distribution with a standard deviation of 20 hours. We sample 25 devices and find their average lifespan to be 230 hours.
  - a. Construct a 90% confidence interval for the battery's average lifespan.
  - b. Will a 95% confidence interval be shorter or longer? Explain.
  
- 4) The average monthly salary of 200 employees in Ohio is found to be \$5,000 . Assume that the population standard deviation of salaries is \$1,000 .
  - a. Construct a 95% confidence interval for the actual average salary.
  - b. What sample size is needed to shorten the confidence interval by 50%?
  - c. If we expand the sample size from 200 and construct a 95% confidence interval, what happens to the interval?
  
- 5) A company is studying the recovery time of a new drug. 60 people participated in the study, with an average recovery of 4 days. The population standard deviation is 2 days.
  - a. Construct a confidence interval for the mean recovery time at a 90% level of confidence.
  - b. What happens to the length of the confidence interval, if the sample were four times as large? Explain.
  - c. What happens to the length of the confidence interval, if we use a higher level of confidence? Explain.

- 6) A researcher constructs a confidence interval for an average, using a sample of 16 observations and obtains  $82 < \mu < 92$ .

Assume that the variable follows a normal distribution and that its standard deviation is 10.

- a. What is the sample average?
  - b. What is the level of confidence for this confidence interval?
  - c. What are the chances that the estimation error is greater than 5?
- 7) Which of the following factors does not affect the length of the confidence interval (when the population variance is known)?
- a. The confidence level.
  - b. The population standard deviation.
  - c. The sample size.
  - d. The sample standard deviation.
- 8) A researcher constructs a confidence interval for the average and obtains the following confidence interval:  $63 < \mu < 83$ .
- Assume a known population standard deviation, and a sample size of 40.
- a. What sample size is needed for a confidence interval of length 10?
  - b. The original confidence level was 95%. Construct a confidence interval at a 98% level of confidence.



## Unknown Population Variance

---

### Questions

- 1) A study investigates how a certain drug affects pulse rate. A sample of 5 participants measured their pulse and recorded the number of beats per minute: 89, 79, 84, 88, 84. Assume that pulse rate is approximately normal.
  - a. Construct a 95% confidence interval for the expected pulse rate among all users of this drug.
  - b. Assuming that the average pulse rate for people who do not take the drug is 70, does the drug affect pulse rate, at a 95% level of certainty?
  - c. In continuation of part a: if we were to construct a confidence interval at a 99% level of certainty instead, what will happen to the confidence interval?
  
- 2) In a sample of 25 college students, the average height was 178cm, with a standard deviation of 13cm.  
Create a confidence interval at a 90% level of confidence for the expected height of college students.
  
- 3) Steve wants to estimate the average time (in minutes) that it takes him to get to work. He samples his commute time for five days, with the following results: 27, 34, 32, 40, 30.
  - a. Estimate the average travel time at a 95% level of certainty.
  - b. How would the size of the confidence interval change, if Steve sampled more days?
  
- 4) Scores on an intelligence test follow a normal probability distribution. The scores of 25 people averaged 102, with a sample standard deviation of 13.
  - a. Construct a confidence interval for the population average score at a 95% level of certainty.
  - b. Repeat part a, assuming that the obtained standard deviation of 13 is the population standard deviation.
  - c. Explain the differences in the answers to parts a and b.
  
- 5) 60 babies were weighed at birth, with an average of 7.7 lbs and a sample standard deviation of 1 lb.  
Construct a confidence interval for birth weight at 95% confidence.  
Explain what this means.

- 6) Two statisticians constructed 95% confidence intervals for the same parameter  $\mu$ . Each statistician had a different sample of 10 observations. Statistician A assumed that  $\sigma = 20$ . Statistician B calculated the sample standard deviation, and found that  $\sigma = 20$ . Which of the statisticians will have a longer confidence interval (select the correct answer)?
- Statistician A.
  - Statistician B.
  - Both statisticians will have confidence intervals of the same length.
  - It depends on the sample results of each statistician.

### Answer Key:

- 1) a. (79.88, 89.72);                      b. Yes.                      c. Interval will be longer.
- 2) (173.55, 182.45)
- 3) a. (26.543, 38.657);                      b. Interval will be shorter.
- 4) a. (96.634, 107.37);                      b. Interval will be shorter.
- 5) (7.4417, 7.9583)
- 6) b. Statistician B.