

# Workbook



## Table of Contents

Differentiability .....	2
Differentiability of Piecewise Functions .....	2

# Differentiability

## Differentiability of Piecewise Functions

### Questions

- 1) Determine points of non-differentiability of the following functions, and find  $f'(2)$ , if possible.

$$\text{a. } f(x) = \begin{cases} x^2 - 4x & x \geq 2 \\ x^3 - 14 & x < 2 \end{cases}$$

$$\text{b. } f(x) = \begin{cases} x^2 - 5x & x \geq 2 \\ x^3 - 14 & x < 2 \end{cases}$$

$$\text{c. } f(x) = \begin{cases} x^2 + 8x & x \geq 2 \\ x^3 + 12 & x < 2 \end{cases}$$

- 2) Determine points of non-differentiability of the following functions, and write the formula for  $f'(x)$ :

$$\text{a. } f(x) = \begin{cases} \ln(1+2x) & -0.5 < x < 0 \\ x^2 + 2x & x \geq 0 \end{cases}$$

$$\text{b. } f(x) = 2 + 4|x-1|$$

$$\text{c. } f(x) = 3x^2 + x|x| + 1$$

- 3) Determine points of non-differentiability of the following functions, and find  $f'(0)$  if possible:

$$\text{a. } f(x) = \begin{cases} x \cdot \sin \frac{1}{x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\text{b. } f(x) = \begin{cases} x^2 \cdot \sin \frac{1}{x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- 4) For what value or values of the constants  $a$  and  $b$ , is  $f$  differentiable for all values of  $x$ :

$$\text{a. } f(x) = \begin{cases} x^2 + ax & x \geq 2 \\ x^3 + b & x < 2 \end{cases}$$

$$\text{b. } f(x) = \begin{cases} e^x & x \leq 0 \\ ax + b & x > 0 \end{cases}$$

$$\text{c. } f(x) = \begin{cases} \ln^3 x & x \geq e \\ ax + b & x < e \end{cases}$$

**Answer Key**

1) a.  $x = 2$

b.  $x = 2$

c.  $f(x)$  is differentiable for all  $x$ .

2) a.  $f'(x) = \begin{cases} \frac{2}{1+2x} & -0.5 < x < 0 \\ 2 & x = 0 \\ 2x+2 & x > 0 \end{cases}$

b.  $x = 1$

c.  $f(x)$  is differentiable for all  $x$ ,  $f'(x) = \begin{cases} 8x & x > 0 \\ 0 & x = 0 \\ 4x & x < 0 \end{cases}$

3) a.  $f'(x) = \begin{cases} \sin \frac{1}{x} - \frac{1}{x^2} x \cos \frac{1}{x} & x > 0 \\ \text{not exist} & x = 0 \\ 0 & x < 0 \end{cases}$

b.  $f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x} & x > 0 \\ 0 & x = 0 \\ 0 & x < 0 \end{cases}$

4) a.  $a = 8, b = 12$

b.  $b = 1$

c.  $a = \frac{3}{e}, b = -2$ .