proprep™

Workbook

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Data Distributions

Transformations of Measuring Scales

Questions

1) State for which measuring scales the following transformations are permitted:
  a. Multiplying by zero.
  b. Multiplying by 2.
  c. Multiplying by -1.
  d. Adding 3.
  e. Subtracting 3.

<u>Select the most correct answer in the following questions:</u>

2) Which transformation preserves the measuring scale of the variable “The Temperature in the Oval Room”?
  a. A transformation that preserves order.
  b. A positive linear transformation.
  c. A transformation that preserves ratio.
  d. Answers b and c are correct.

3) In which measuring scale is subtracting a constant from each number permitted?
  a. Only a nominal scale.
  b. Only a nominal or ordinal scale.
  c. Only a nominal, ordinal, or interval scale.
  d. In all four measuring scales.

4) Which transformation preserves the scale of bus numbers?
  a. A transformation that preserves order.
  b. A positive linear transformation.
  c. A transformation that preserves ratio.
  d. All the answers are correct.

Answer Key

1) a. No scale b. All scales c. Nominal d+e. All but ratio.

2) d **3)** c **4)** d

Measures of Relative Location - Standard Score

Questions

1) 8^{th} grade students take exams in English and Math.
The following results were obtained:

a. Mike received a 68 in English and a 70 in Math. In which subject is Mike better, relative to the rest of his class?

b. What mark did Mike need in Math to be the equivalent of his mark in English?

Subject	Average	Standard Deviation
English	74	12
Math	80	16

2) In a car battery factory, we count the number of employees per day and measure their daily output (the number of batteries – in hundreds) for 40 continuous days.
The following table shows the information collected about the two variables:
On one of the 40 days, 13 employees were working and produced 5,000 (50 hundred batteries).
What was more exceptional about that day, relative to the other days on which output and the Number of employees were examined?
Select the correct answer:

a. Output

b. Number of employees

c. Equally exceptional

d. Data needed to draw a conclusion are lacking.

	Output	No. of Employees
Average	48	15
Standard deviations	10	2

3) The average height of people drafted to the army is 175cm, with a standard deviation of 10cm. The average weight is 66kg with a standard deviation of 8kg.
Steve, a recruit, is 180cm high and weighs 59kg.

a. In what does Steve deviate more in comparison with the other recruits: height or weight?

b. How much should Steve weigh to make his weight the equivalent of his height?

Answer Key

1) a. English b. 72

2) b

3) a. Weight b. 70

Measures of Relative Location - Percentiles in Categories

Questions

1) The following table displays the probability distribution of the salaries (in $\$k$) in a company:

a. Calculate the 60^{th} percentile.

b. What is the top 10^{th} percentile (90^{th} percentile)?

c. The highest 20% of the salaries are earned by senior employees. What is the minimum salary for a senior employee?

d. What percentage of the employees earns less than $\$7,000$?

e. What percentage of the employees earns more than $\$25,000$?

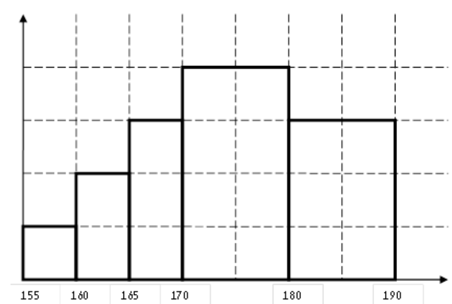
f. What percentage of the employees earns between $\$7,000$ and $\$25,000$?

X - Salary	Cumulative Frequency
6-10	48
10-15	100
15-20	120
20-30	132
30-60	136

2) 400 people take an exam. It is known that the bottom 10^{th} percentile is the mark 60, the top quartile is the mark 80, and the probability distribution of the marks is symmetric.
Fill in the missing frequencies.

X - Mark	$f(x)$
50-60	
60-70	
70-80	
80-90	
90-100	

3) The following histogram describes the probability distribution of the height (in cm) in a given group.
Calculate:



a. The bottom 10^{th} percentile.

b. The 30^{th} percentile.

c. The value over which 20% of the observations are found

d. The percentage of the observations below 158cm.

e. The percentage of the observations above 185cm.

f. The percentage of the observations between 170cm and 185cm.

Answer Key

1) a. \$13.23K b. \$22K c. \$17.2K d. 8.82%

e. 7.36% f. 83.82%

2)

X - Mark	$f(X)$
50-60	40
60-70	60
70-80	200
80-90	60
90-100	40

3) a. 162.5 b. 170 c. 183.33 d. 3% e. 15% f. 55%

Percentiles in a Discrete Frequencies Table

Questions

1) The following table displays the probability distribution of a discrete variable:
For this probability distribution, find:

a. The 60^{th} percentile.
b. The 40^{th} percentile.
c. The top 10^{th} percentile (90^{th} percentile).
d. The inter-quartile range.

X	$f(X)$	$F(X)$	$\frac{F(X)}{n}$
0	10		
1	40		
2	30		
3	15		
4	5		

2) The following table displays the number of cars owned by family in a certain community:
Calculate:

a. The bottom 10^{th} percentile.
b. The 30^{th} percentile.
c. The value which 20% of the observations are greater than.
d. The upper quartile.

No. of cars in the family	1	2	3	4	5
$f(X)$	65	150	220	140	55
$F(X)$					
$\frac{F(X)}{n}$					

Answer Key

1) a. 2 b. 1 c. 3 d. 1

2) a. 1 b. 2 c. 4 c. 4

Linear Transformations

Questions

1) For a certain series of data: $\overline{X}=80$, $S=15$, $MO=70$.
It is decided to multiply every observation by 4 and to subtract 5 from the result.
Calculate the above meaures after the transformation.

2) The average salary in a certain company is $\$40$ per hour, with a standard deviation of $\$5$ per hour. Management decided to raise all the salaries by 10%, but this did not satisfy the employees, therefore they received a raise of an additional $\$2$ per hour.
What are the average and variance of the hourly rate after all the changes?

3) The median mark in an exam is 73, the range of the marks is 40 points, and the upper 10^{th} percentile is 87. Because the students' marks on the exam were low, the teacher decided to boost the students' mark by 4 points.
Calculate the median, range and the upper 10^{th} percentile after the factor.

4) 50 matchboxes are sampled from a production line producing matchboxes containing 40 matches per box. The number of faulty matches is checked in vary. An average of 3 faulty matches were found per matchbox, with a standard deviation of 1.
What are the average and standard deviation of the number of good matches in a box?

5) Bell Telecommunications has offered the following package: $\$30$ fixed monthly subscription fees, plus $\$0.10$ for each outgoing call per minute.
A person checked his outgoing calls for a year, and found that he averaged 600 minutes of outgoing calls per month, with a variance of 2,500 rounded off minutes. In January, his standard score was 2.
Calculate the average, variance ad standard score for this subscriber's monthly telephone bill (in $\$$) if he used the packaged proposed to him by Bell.

6) Prove that if all the observations in a probability distribution have undergone a linear transformation: $Y_i = a \cdot X_i + b$
then the average and variance of all the observations after the transformation will be:
$\overline{y} = a \cdot \overline{x} + b, \quad s^2{}_y = a^2 s^2{}_x$

Answer Key

1) Average: 315, Standard deviation: 60, Mode: 275.

2) Average: $46\ \$/\text{h}$, Variance: $30.25\ \$^2/\text{h}$.

3) Range: 40, Median: 77, Upper 10^{th} percentile: 91.

4) Average: 37, Standard deviation: 1.5.

5) Average: $\$90$, Variance: $\$25^2$, Standard score: 2.

6) a. $\bar{y} = \dfrac{\sum y}{n} = \dfrac{\sum(ab+b)}{n} = \dfrac{a\sum x}{n} + \dfrac{nb}{n} = a\bar{x} + b$

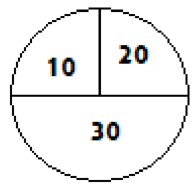
b. $s^2{}_y = \dfrac{\sum(y_i - \bar{y})}{n} = \dfrac{\sum\left[(a \cdot x_i + b) - (a \cdot \bar{x} + b)\right]^2}{n} = \dfrac{\sum a^2 (x_i - \bar{x})^2}{n} = a^2 s^2{}_x$

The Probability Function

Summary

- A discrete random variable is a variable that can receive individual values with various probabilities.
- The random variable is described by a probability function.
- A probability function is a function that assigns a probability to each possible value of the random variable.
- The sum of the probabilities of the probability function must be equal to 1.

For **example**, a casino has a roulette wheel as appearing in the following diagram:



A person turns the roulette wheel and wins the amount listed on the roulette, in $\$$.
Construct the probability function of the amount won in a single game.

X	10	20	30
$P(X)$	.025	.025	.05

Questions

1) The distribution of cars in a given community is as follows:
50 families have no car.
70 families have one car.
60 families have two cars.
20 families have three cars.
A family is randomly selected from the community.
We define X as the number of cars owned by the selected family.
Construct the probability function of X.

2) A two-letter code is composed from the letters A, B, and C.

a. How many codes can be composed?
b. List all the possible codes.
c. We define X as the number of times that B appears in the code.
Construct the probability function of X.

3) A student takes semester exams in economic and statistics.
The chances of passing the economics exam are 0.8, and the chances of passing the statistics exam are 0.9. The chances of passing both exams are 0.75.
Let X be the number of exams that the student passes.
Construct the probability function of.

4) The chances of winning a certain game are 0.3.
A person plays the game until he wins, but in any case, does not play the game more than four times. We define X as the number of times he plays the game.
Construct the probability function of X .

5) A project management company manages three projects simultaneously.
The chances of Project A succeeding are 0.7, the chances of Project B succeeding are 0.8, and the chances of Project C succeeding are 0.9.
The success of each project is independent of the other projects.
We define X as the number of successful projects.
Construct the probability function of X .

6) The following is the probability function of a given random variable: $P\left(X=k\right)=\frac{k}{A}$
where $k=1,2...4$, Find the value of A .

7) A kindergarten has eight children - five boys and three girls.
Three children are randomly selected to take part in a play.
We define X as the number of boys selected to take part in the play.
Construct the probability function of X .

8) A survey examined whether people watched the news broadcasts of Channels 1, 2, and 3. The following information was discovered: 20% of viewers watch Channel 2. 8% of viewers watch Channel 1. 10% of viewers watch Channel 3. 1% of viewers watch all three Channels together. 10% of viewers watch two of the three channels. We define X as the number of news broadcasts watched by a randomly selected viewer.
Construct the probability function of X .

Answer Key

1) Answer in the following Table:

X	0	1	2	3
$P(X)$	0.25	0.35	0.3	0.1

2) a. 9. b. $\Omega = \{AA, AB, AC, BA, BB, BC, CA, CB, CC\}$
c. Answer in the following table:

X	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

3) Answer in the following table:

X	0	1	2
$P(X)$	0.05	0.2	0.75

4) Answer in the following table:

X	1	2	3	4
$P(X)$	0.3	0.21	0.147	0.343

5) Answer in the following table:

X	0	1	2	3
$P(X)$	0.006	0.092	.0498	0.504

6) 10.

7) Answer in the following table:

X	0	1	2	3
$P(X)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

8) Answer in the following table:

X	0	1	2	3
$P(X)$	0.74	0.15	0.1	0.01

Expectation, Variance and Standard Deviation

Questions

1) A person plays a game of chance.
We define X as the amount he wins.
The probability function of X is as follows:

X	-30	0	20	40
$P(X)$	0.4	0.1	0.3	0.2

What are the expectation, variance, and standard deviation of X ?

2) There are two bank branches in a given community: City Bank and Union Bank. 50% of the population have an account at City Bank, and 40% have an account at Union Bank. 20% of the population have no bank account.
Let X be the number of bank branches in which a person has an account.
Calculate $E(X)$.

3) It is known that 20% of families have a satellite TV connection in their homes.
In a survey, a pollster seeks to interview a family with a satellite TV connection.
He randomly telephones a family and continues until he finds a family with a satellite TV connection. In any case, the pollster does not call more than five families.
We define X as the number of families called in the survey.

a. Construct the probability function of X .
b. Calculate the expectation and standard deviation of X .

4) A person has a key ring with five keys, only one of which opens the door to his home.
He tries the keys randomly.
After trying a given key, he takes it off the ring in order to avoid using it again.
Let X denote the number of attempts until the door opens.

a. Construct the probability function of X .
b. Calculate the expectation and variance of X .

5) The probability function of X , a random variable, is as follows:
It is given that $E(X)=4.2$.

a. Find the missing probabilities in the table.
b. Calculate $V(X)$.

X	2	4	6	8
$P(X)$		0.3		0.2

6) A discrete random variable receives the values -5, 0, and 5.
The variable's expectation is 0, and its variance is 10.
Find the probability function.

7) The probability distribution of X, a random variable, is as follows:
What value of k will give the minimum value for the variance of X ?

X	1	3	k
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Answer Key

1) $E(X)=2$, $V(X)=796$, $\sigma(X)=28.21$

2) $E(X)=0.9$

3) a.

X	1	2	3	4	5
$P(X)$	0.2	0.16	.0128	0.1024	.04096

b. $E(X)=3.3616$, $V(X)=2.57$, $\sigma(X)=1.603$

4) a.

X	1	2	3	4	5
$P(X)$	0.2	0.2	0.2	0.2	0.2

b. $E(X)=3$, $V(X)=2$

5) a.

X	2	4	6	8
$P(X)$	0.4	0.3	0.1	0.2

b. $V(X)=5.16$

6)

X	-5	0	5
$P(X)$	0.2	0.6	0.2

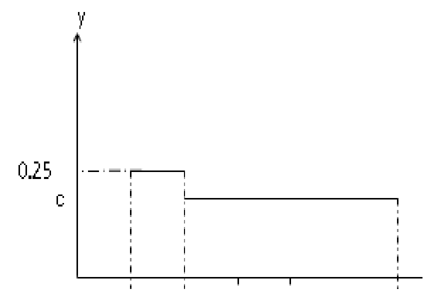
7) $K=2.33$

General Probabilities without Integrals

Questions

1) X is a continuous variable having the density function shown below:

a. Find the value of c.
b. Construct the cumulative distribution function.
c. Calculate the following probabilities:
   i. $P(X=4)$
   ii. $P(X>1.5)$
   iii. $P(1.5<X<5)$
   iv. $P(5<X<10)$
d. Find the median of X.



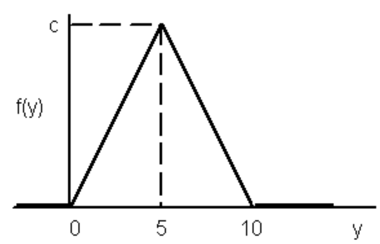
2) A continuous random variable X has the following density function:
$f(x)=cx$, where $0 \le x \le b$, 0 otherwise.

It is known that $P(0<X<1)=\frac{1}{4}$.

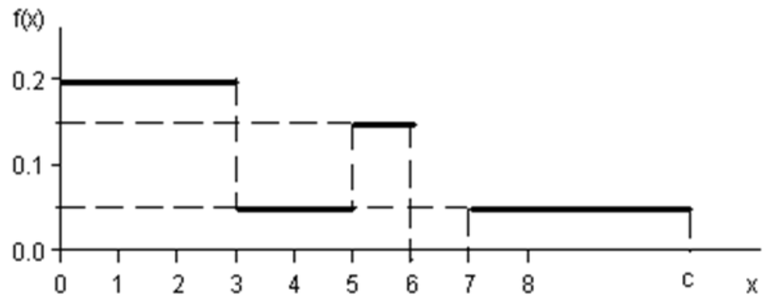
a. Write the density function of X.
b. Find the median of X.
c. What are the chances of X being less than 0.5?

3) The diagram below shows the density function of the random variable Y.

a. Calculate c.
b. Write the cumulative distribution function of Y.
c. Calculate the probabilities: $P(Y>4)$, $P(7.5<X<15.5)$, $P(Y\le 3)$, $P(Y=7)$.
d. Calculate the bottom 10^{th} percentile $Y_{0.1}$, the bottom 25^{th} percentile $Y_{0.25}$, and the median of Y and the upper 10^{th} percentile $Y_{0.9}$.



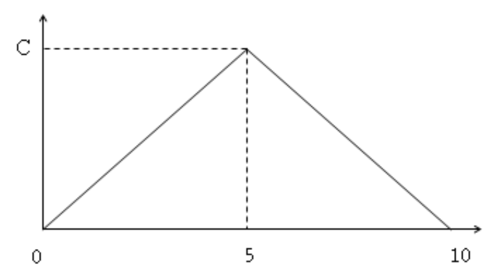
4) The diagram below shows the density function of the random variable X :



a. Calculate the value of c that define the diagram as a density function.
b. Write the cumulative distribution function of X .
c. Calculate the probabilities: $P(1<X\leq 5)$, $P(X\geq -2)$, $P(X\geq 4)$.

5) Given the following density function

a. What is the value of c ?
b. Find a symmetric interval around the value 5 in which the probability equals 0.5.



6) The waiting time in minutes of a customer in line at the neighborhood supermarket has the following cumulative distribution function: $F(t)=1-e^{-0.2t}$.

a. What are the chances that the waiting time will be less than 15 minutes?
b. What is the probability of a customer waiting in line a total of less than 15 minutes if he has already waited in line for 10 minutes?
c. What is the time under which 90% of the customers have to wait?

Answer Key

1) a. $\frac{3}{16}$

b. $F(t)=P(x\le t)=\begin{cases}0 & x<1\\ 0.25(t-1) & 1\le x\le 2\\ 0.25+\frac{3}{16}(t-1) & 2<x<6\\ 1 & x>6\end{cases}$

c.(i) $\frac{5}{8}$ (ii) $\frac{7}{8}$ (iii) $\frac{11}{16}$ (iv) $\frac{3}{16}$ d. $3\frac{1}{3}$

2) a. $f(x)=\begin{cases}\frac{1}{2}x & 0<x\le 2\\ 0 & \text{other}\end{cases}$ b. 1.41 c. $\frac{1}{16}$

3) a. $\frac{1}{5}$

b. $F(t)=P(y\le t)=\begin{cases}0 & y<0\\ 0.02t^2 & 1\le y\le 5\\ 1\text{-}(t-10)^2\cdot 0.02 & 5<y\le 10\\ 1 & y>10\end{cases}$

c. $\mathrm{P}(y=7)=0$; $\mathrm{P}(y\le 3)=0.18$; $\mathrm{P}(7.5<y<15.5)=0.125$ $\mathrm{P}(y>4)=0.32$

d. $P(Y\le y_{0.1})=0.1\Rightarrow t=\sqrt{5}$; $P(Y\le y_{0.25})=0.25\Rightarrow t=\sqrt{12.5}$

$Y_{0.9}=7.76$; $\text{median}=5$

4) a. $c=10$

b. $F(t)=\begin{cases}t<0 & 0\\ 0\le t\le 3 & 0.2\text{t}\\ 3<t\le 5 & 0.6+0.05(t-3)\\ 5<t\le 6 & 0.7+0.15(t-5)\\ 6<t\le 7 & 0.85\\ 7<t\le 10 & 0.85+0.05(t-7)\\ t>10 & 1\end{cases}$

c. $P(X\ge 4)=0.35$; $P(X\ge -2)=1$; $P(1\le X\le 5)=0.5$

5) a. $c=0.2$ b. 5 ± 1.46

6) a. $P(x\ge 15)=0.0498$ b. $P(x<15|x>10)=0.6321$ c. $t=x_{0.9}=115.13$

Normal Probability

Questions

1) The height of people in a given population has a normal probability distribution with an average of 170cm and a standard deviation of 10cm.

a. What is the proportion of people who are shorter than 182.4cm?
b. What is the proportion of people who are taller than 190cm?
c. What is the proportion of people who are exactly 173.6cm tall?
d. What is the proportion of people who are shorter than 170cm?
e. What is the proportion of people who are at most 170cm tall?

2) Assume that the time it takes for a certain medication to take effect has a normal probability distribution with an average of 30 minutes and a variance of nine minutes.

a. What is the proportion of cases where the medication takes longer than an hour to work?
b. What is the proportion of cases where the medication takes between 35 and 37 minutes to work?
c. What are the chances that the medication will help after exactly 36 minutes?
d. What is the proportion of cases where the time taken by the medication to work deviates from 30 minutes by less than three minutes?

3) The weight of people in a given population has a normal probability distribution with an average of 60kg and a standard deviation of 8kg.

a. What is the proportion of people who weigh less than 55kg?
b. What is the proportion of people in the population who weigh less than 50kg?
c. What is the relative frequency of the people in the population who weigh between 60 and 70kg?
d. What is the proportion of the population whose weight deviates from the average by no more than 4kg?
e. What are the chances of a randomly selected person weighing less than 140kg?

4) The weight of babies at birth has a normal probability distribution with an average of 3300 grams and a standard deviation of 400 grams.

a. Find the upper 10^{th} percentile.
b. Find the 95^{th} percentile.
c. Find the bottom 10^{th} percentile.

5) Marks in an intelligence test have a normal probability distribution with an average of 100 and a variance of 225.

a. What is the upper 10^{th} percentile of the marks on the intelligence test?
b. What is the bottom 10^{th} percentile of the probability distribution?
c. 20% of those taking the test receive marks higher than what number?
d. What is the 20^{th} percentile?
e. 5% of those taking the test receive marks lower than what number?

6) The volume of a bottled beverage has a normal distribution with a standard deviation of 20ml. Assume that 33% of the bottles have a volume of over 508.8ml.

a. What is the average volume of a bottled beverage?
b. 5% of the bottles that are produced with the largest volume are sent for testing. Starting from what volume are bottles sent for testing?
c. 1% of the bottles with the lowest volume are donated to charity. What is the maximum volume of the bottles donated to charity?

7) The lifespan of a device has a normal probability distribution. It is known that half of the devices last less than 500 hours, and that 67% of the devices last less than 544 hours.

a. What is the average lifespan of a device?
b. What is the standard deviation of the lifespan of a device?
c. What are the chances that a randomly selected device will last less than 460 hours?
d. What is the upper 1 percentile of a device's lifespan?
e. 1% of the devices with shortest lifespan are sent to the laboratory for a thorough check. What is the maximum lifespan of a device sent to the laboratory?

8) The following are three normal probability distributions, of three different groups, sketched on system of coordinate axes:

a. Which probability distribution has the highest average?
b. In which of the following measures are distributions 1 and 2 the same?
   i. In their upper 10^{th} percentile.
   ii. In their average.
   iii. In their variance.

c. Which distribution has the smallest standard deviation?
   i. 1
   ii. 2
   iii. 3
   iv. No option.

9) The time it takes a person to get to work has a normal probability distribution with an average of 40 minutes and a standard deviation of five minutes.

a. What is the probability that it takes less than 45 minutes for a person to get to work?
b. A person leaves home to go to work at $8:10$. He has to get there by $9:00$. What are the chances of him being late?
c. If it is known that it takes a person longer than 45 minutes to get to work, what is the probability that the total time it took him is less than 50 minutes?
d. What are the chances it will take a person at least 45 minutes to get to work at least once during a five-day work week?

10) The monthly household spending in the city of Tarera has a normal probability with an average of $\$2,000$ and a standard deviation of $\$300$. Five households are randomly selected. The probability that at least one of them spends more than T dollars per month is 0.98976.

a. What is the value of T?
b. What are the chances that a household in the town spends at least one standard-deviation more than T?
c. It is learned that a mistake was made in the data, and $\$100$ must be added to the monthly spending of all the households in the city.
Given this correction, what is the probability that a household's monthly spending is less than $\$1,800$?

11) The length of a random song that is broadcasted on the radio has a normal probability distribution with an expectation of 3.5 minutes and a standard deviation of 30 seconds.

a. What is the probability that the length of a random song played on the radio is between 2.5 and 3 minutes?
b. What is the inter-quartile range of the length of a song broadcast on the radio?
c. 200 songs are played on the radio on a given day.
How many songs shorter than 3.5 minutes can we expect to be played?
d. Eight songs are broadcasted during a given hour.
What is the probability that exactly a quarter of them were longer than four minutes, and the rest were no longer?

Answer Key

1)	a. 89.25%	b. 2.28%	c. 0	d. 50%	e. 50%
2)	a. 0%	b. 3.76%	c. 0	d. 68.26%	
3)	a. 26.43%	b. 89.44%	c. 39.44%	d. 38.3%	e. $\cong 1$
4)	a. 3812.8	b. 3958	c. 2787.2		
5)	a. 119.23	b. 80.77	c. 112.6	d. 87.4	e. 75.325
6)	a. 500	b. 532.9	c. 453.48		
7)	a. 500	b. $\sigma = 100$	c. 0.3446	d. 732.6	e. 267.4
8)	a. 3	b. In their average.	c. 1		
9)	a. 0.1587	b. 0.0228	c. 0.1359	d. 0.3975	
10)	a. $T = 1925$	b. 0.2266	c. 0.1587		
11)	a. 0.1359	b. 0.675	c. $\mathrm{E}(y) = 100$		d. 0.25