

Workbook



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Taylor and Maclaurin Series

Basic Exercises with Maclaurin Series

Questions:

- 1) Find the Taylor series of $f(x) = \sin 2x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 2) Find the Taylor series of $f(x) = x^2 e^{-4x}$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 3) Find the Taylor series of $f(x) = \sinh x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 4) Find the Taylor series of $f(x) = \sin^2 x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 5) Find the Taylor series of $f(x) = \cos^2 x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 6) Find the Taylor series of $f(x) = 2^x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 7) Find the Taylor series of $f(x) = x \cos(4x^2)$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 8) Find the Taylor series of $f(x) = \ln(2 - 3x + x^2)$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix)
- 9) Find the Taylor series of $f(x) = \arcsin x$ around $x = 0$.
(Use the table of Maclaurin expansions in the Appendix).

Expansions about General Point

Questions:

- 10) Find the Taylor series of $f(x) = \ln x$ expanded around $x = 1$.
- 11) Find the Taylor series of $f(x) = \frac{1}{x}$ expanded around $x = 2$.
- 12) Find the Taylor series of $f(x) = \sin x$ expanded around $x = \frac{\pi}{2}$.

Finding Nonzero Terms in Expansions

Questions:

- 13) Find the first four nonzero terms of the Maclaurin series of $f(x) = e^{-x^2} \cos x$.
- 14) Find the first four nonzero terms of the Maclaurin series of $f(x) = \tan x$.
- 15) Find the first four nonzero terms of the Maclaurin series of $f(x) = \frac{\sin x}{e^x}$.

Sum of Series Using Taylor and Maclaurin Expansions Questions:

16) Compute the sum of the following series:

a. $\sum_{n=0}^{\infty} \frac{1}{n!}$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

c. $\sum_{n=0}^{\infty} \frac{1}{2^n \cdot n!}$

17) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{n+1}{n!}$.

18) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

19) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$.

20) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$.

21) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$.

22) Compute the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(n+1)}$.

Finding Limits Using Expansions

Questions:

23) Compute the value of the following limit:

- $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x^3}$
- $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$
- $\lim_{x \rightarrow 0} \frac{e^x \sin x - x(1+x)}{x^3}$

Computations with Taylor Series

Questions:

24) Answer the following questions:

- Compute $1/\sqrt[3]{e}$ with an error of less than 0.001.
- Compute $\sin 3^\circ$ with an error of less than 0.001.
- Compute $\arctan 0.25$ with an error of less than 0.001.

25) Answer the following questions:

- Evaluate $\frac{1}{\sqrt{e}}$ using the first three nonzero elements of maclaurin series and estimate the error.
- Evaluate $\cos 4^\circ$ using the first three nonzero elements of maclaurin series and estimate the error.
- Evaluate $\ln 1.5$ using the first three nonzero elements of maclaurin series and estimate the error.

26) Answer the following questions:

- What is the maximum error in approximating $\sin x \cong x - \frac{x^3}{3!}$ for $|x| \leq \frac{\pi}{6}$?

- b. What is the maximum error in approximating $\ln(1+x) \cong x$ for $|x| < 0.01$?
- c. What is the maximum error in approximating $\cos x \cong 1 - \frac{x^2}{2!}$ for $|x| \leq 0.2$?

27) Answer the following questions:

- a. For which values of x is $\sin x \cong x - \frac{x^3}{3!}$ with error less than 0.001?
- b. For which values of x is $\arctan x \cong x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$ with error less than 0.01?

28) Answer the following questions:

- a. Approximate $\int_0^{0.2} \frac{\sin x}{x} dx$ with an error less than 0.0001.
- b. Approximate $\int_0^{0.1} \frac{\ln(1+x)}{x} dx$ with an error less than 0.001.
- c. Approximate $\int_0^{0.5} \frac{1-\cos x}{x^2} dx$ with an error less than 0.001.

Answer Key:

- 1) $\sum_{n=0}^{\infty} (-1)^n 2^{2n+1} \frac{x^{2n+1}}{(2n+2)!}$ 2) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{n!}$ 3) $\sinh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ all x
- 4) $\sum_{n=0}^{\infty} (-1)^{n+1} 2^{2n-1} \frac{x^{2n}}{2n!}$ 5) $\frac{1}{2} + \sum_{n=0}^{\infty} (-1)^n 2^{2n-1} \frac{x^{2n}}{(2n)!}$ 6) $\sum_{n=0}^{\infty} (\ln 2)^n \frac{x^n}{n!}$
- 7) $\sum_{n=0}^{\infty} (-1)^n 4^{2n} \frac{x^{4n+1}}{(2n)!}$ all x 8) $\ln 2 - \sum_{n=0}^{\infty} \left(1 + \frac{1}{2^{n+1}}\right) \frac{x^{n+1}}{n+1}$ $-1 \leq x < 1$
- 9) $x + \sum_{n=0}^{\infty} (-1)^n \frac{\frac{1}{2} \left(\frac{1}{2} - 1\right) \dots \left(\frac{1}{2} - n + 1\right)}{n!} \frac{x^{2n+1}}{2n+1}$ $-1 \leq x \leq 1$
- 10) $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$ 11) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$ $0 < x < 4$
- 12) $\sum_{n=0}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!}$ all x 13) $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 - \frac{331}{720}x^6$
- 14) $x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7$ $-\frac{\pi}{2} < x < \frac{\pi}{2}$ 15) $x - x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$
- 16) a. e b. $\frac{1}{e^2}$ c. \sqrt{e} 17) $2e$
- 18) $\frac{\pi}{4}$ 19) $\sin 1^\circ$ 20) $\cos 1^\circ$ 21) $\ln 2$
- 22) $\ln 1.5$ 23) a. $\frac{1}{3}$ b. $\frac{1}{120}$ c. $\frac{1}{3}$
- 24) a. $\frac{58}{81}$ b. $\frac{\pi}{60}$ c. $\frac{47}{192}$
- 25) a. $\frac{5}{8}$ b. 1 c. $\frac{77}{192}$
- 26) a. $\frac{\left(\frac{\pi}{6}\right)^5}{5!}$ b. 0.00005 c. $\frac{0.2^6}{720}$
- 27) a. $|x| < \sqrt[3]{0.12}$ b. $|x| < \sqrt[3]{0.09}$
- 28) a. $\frac{449}{2259}$ b. $\frac{39}{400}$ c. $\frac{143}{576}$