

Workbook



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The Derivative of a Function

Basic Derivatives of Functions

Questions

1) Find the first derivative:

- a. $f(x) = 4$ b. $g(x) = \frac{e + \sqrt{2}}{2}$ c. $h(x) = x^4$
- d. $y = \frac{1}{x^2}$ e. $f(x) = \sqrt{x}$ f. $y = \frac{1}{\sqrt{x}}$
- g. $y = 4x^{10} + \frac{1}{x}$ h. $y = x + x^2$ i. $y = 4x^2 + 8x^3 - 5$
- j. $y(x) = \frac{2}{x} - ex$ k. $y = \sqrt{2}x^2 + 2ex$ l. $y(t) = \frac{4}{t} + \sqrt[3]{t}$

2) Find the first derivative:

- a. $y = (x^2 + 3)(x - 1)$ b. $y = (4x + 10)(\sqrt{x} - 1)$
- c. $y = (x - 1)(x - 1)(x - 2)$ d. $y = \frac{4x + 10}{x^2 - x}$
- e. $y = \frac{x^2 + 4x - 1}{2x - 3}$ f. $y = \frac{ex + 1}{ex - 1}$
- g. $y = (4x + 10)^3$ h. $y = (x^2 + 1)^5$
- i. $y = \sqrt{(x^2 + x + 1)^3}$ j. $y = (2x + 1)^3(4x - 5)^4$
- k. $y = \frac{(2x + 3)^4}{(x - 5)^3}$ l. $y = \frac{1}{\sqrt[3]{4x + 1}}$

Answer Key

1) a. $f'(x) = 0$

b. $g'(x) = 0$

c. $h'(x) = 4x$

d. $y'(x) = \frac{-2}{x^3}$

e. $f'(x) = \frac{1}{2\sqrt{x}}$

f. $z'(x) = -\frac{1}{2x^{1.5}}$

g. $y'(x) = 40x^9 - \frac{1}{x^2}$

h. $y'(x) = 1 + 2x$

i. $y'(x) = 8x + 24x^2$

j. $y'(x) = -\frac{2}{x^2} - e$

k. $y'(x) = 2\sqrt{2x} + 2e$

l. $y'(t) = -\frac{4}{t^2} + \frac{1}{3\sqrt[3]{t^2}}$

2) a. $y'(x) = 3x^2 - 2x + 3$

b. $y'(x) = 6\sqrt{x} - 4 + \frac{5\sqrt{x}}{x}$

c. $y'(x) = 3x^2 - 8x + 5$

d. $y'(x) = \frac{2(2x^2 + 10x - 5)}{x^2(x-1)^2}$

e. $y'(x) = \frac{2(x^2 - 35 - 5)}{(3 - 2x)^2}$

f. $y'(x) = \frac{-2e}{(ex-1)^2}$

g. $y'(x) = 12(4x+10)^2$

h. $y'(x) = 10x(x^2 + 1)^4$

i. $y'(x) = \left(3x + \frac{3}{2}\right)\sqrt{x^2 + x + 1}$

j. $y'(x) = 14(2x+1)^2(4x-5)^3(2x-1)$

k. $y'(x) = \frac{(2x+3)^3(2x-49)}{(x-5)^4}$

l. $y'(x) = -\frac{4}{3(4x+1)^{\frac{4}{3}}}$

Derivative of Exponents and Logarithmic Functions

Questions

1) Find the first derivative:

a. $y = e^x$

b. $y = 4e^x + 2x^3$

c. $y = e^x(x^2 + x + 4)$

d. $y = \frac{e^x}{x^2 - x}$

e. $y = e^{4x-1} + e^{2x}$

f. $y = e^{-x}(x+1)$

g. $y = \frac{e^{2x} - x}{e^{4+x}}$

h. $y = e^{\sqrt{x}}$

i. $y = \frac{e^{\pi x}}{x-2}$

j. $y = \frac{1}{\sqrt{e^{4x} + 1}}$

k. $y = \sqrt[3]{e^{x^2+1} + 1}$

l. $y = \frac{e^{-x^2}}{x}$

2) Find the first derivative and second derivatives:

a. $y = \frac{\ln x}{x}$

b. $y = \ln^2 x + \frac{1}{\ln x}$

c. $y = e^{2x} \ln(x^2 + 4)$

d. $y = \frac{\ln x}{e^x}$

Answer Key

1) a. $y'(x) = e^x$

b. $y'(x) = 4e^x + 6x^2$

c. $y'(x) = e^x(x^2 + 3x + 5)$

d. $y'(x) = \frac{e^x(x^2 - 3x + 1)}{(x^2 - x)^2}$

e. $y'(x) = 4e^{4x-1} + 2e^{2x}$

f. $y'(x) = -xe^{-x}$

g. $y'(x) = e^{-x-4}(x + e^{2x} - 1)$

h. $y'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

i. $y'(x) = \frac{e^{\pi x}(\pi x - (2\pi + 1))}{(x - 2)^2}$

j. $y'(x) = \frac{-2e^{4x}}{(e^{4x} + 1)\sqrt{e^{4x} + 1}}$

k. $y'(x) = \frac{2e^{x^2}x}{3\sqrt[3]{(e^{x^2} + 1)^2}}$

l. $y'(x) = \frac{e^{-x^2}(2x^2 + 1)}{x^2}$

2) a. $y'(x) = \frac{1 - \ln(x)}{x^2}$, $y''(x) = \frac{2\ln(x) - 3}{x^3}$

b. $y'(x) = \frac{2\ln^2(x) - 1}{x\ln^2(x)}$, $y''(x) = \frac{2\ln^4(x) + 2\ln^3(x) + \ln(x) + 2}{x^2\ln^3(x)}$

c. $y'(x) = 2e^{2x}\left(\ln(x^2 + 4) + \frac{x}{x^2 + 4}\right)$, $y''(x) = \frac{2e^{2x}\left(4x^3 - x^2 + 2(x^2 + 4)^2\ln(x^2 + 4) + 16x + 4\right)}{(x^2 + 4)^2}$

d. $y'(x) = \frac{\left(\frac{1}{x} - \ln(x)\right)}{e^x}$, $y''(x) = \frac{e^x(-x^2\ln(x) - 2x - 1)}{x^2}$

Trigonometric Derivatives

Questions

1) Find the first derivative:

a. $f(x) = \sin x$

b. $g(x) = \sin 4x$

c. $y = \cos(0.5x)$

d. $y = \sin^2 x$

e. $f(x) = \cos^4(5x)$

f. $z(x) = \sqrt{\sin 2x}$

g. $y = \sin x \cos 3x$

h. $y = \frac{\sin x - 1}{\cos 2x + 2}$

i. $y = x^3 \sin 4x$

j. $y(x) = \ln(\cos(x))$

k. $y = e^{\sin 2x} \ln x$

l. $y(t) = \sin(\cos(x))$

Answer key

1) a. $f'(x) = \cos(x)$

b. $g'(x) = 4 \cos(4x)$

c. $y'(x) = -0.5 \sin(0.5x)$

d. $y'(x) = 2 \sin(x) \cos(x)$

e. $f'(x) = -500 \sin^3(x)$

f. $f'(x) = \frac{\cos(2x)}{\sqrt{\sin(2x)}}$

g. $y'(x) = \cos(x) \cos(3x) - \sin(x) \sin(3x) - 3 \sin(x)$

h. $y'(x) = \frac{\cos(x) \cos(2x) + 2 \cos(x) + 2 \sin(x) \sin(2x) - 2 \sin(2x)}{(\cos 2x + 2)^2}$

i. $y'(x) = 3x^2 \sin(4x) + 4x^3 \cos(4x)$

j. $y'(x) = -\tan(x)$

k. $y'(x) = e^{\sin 2x} \left(2 \cos(2x) \ln(x) + \frac{1}{x} \right)$

l. $y'(t) = -\sin(t) \cdot \cos(\cos(t))$

Derivative of Power Functions

Questions

1) Find the first derivative:

a. $y = x^{2x}$

b. $y = x^{\ln x}$

c. $y = (\ln x)^x$

d. $y = (x^2 + 1)^{4x}$

e. $y = x^{x^2+1}$

f. $y = (\sqrt{x})^{\sqrt{2x}}$

g. $y = x^{e^x}$

h. $y = (x^{x^x})$

i. $y = (\sin x)^x$

j. $y = x^{\cos 2x}$

k. $y = (\tan x)^{2x}$

l. $y = (\sin x)^{\ln x}$

Answer key

- 1) a. $y'(x) = 2x^{2x} (\ln(x) + 1)$
- b. $y'(x) = \frac{2e^{\ln^2(x)} \ln(x)}{x}$
- c. $y'(x) = \ln(x)^x \cdot \left(\ln(\ln(x)) + \frac{1}{x} \right)$
- d. $y'(x) = 4(x^2 + 1)^{4x} \cdot \left(\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right)$
- e. $y'(x) = x^{x^2+1} \left(2x \ln(x) + \frac{x^2 + 1}{x} \right)$
- f. $y'(x) = \sqrt{x}^{-\sqrt{2x}} \cdot \frac{1}{2} \left(\frac{\ln(x)}{\sqrt{2x}} + \frac{\sqrt{2x}}{x} \right)$
- g. $y'(x) = x^{e^x} \cdot x \left(\ln(x) + \frac{1}{x} \right)$
- h. $y'(x) = x^{x^x+x} (\ln^2(x) + \ln(x) + x^{x-1})$
- i. $y'(x) = (\sin(x))^x \left[\ln(\sin(x)) + x \cot(x) \right]$
- j. $y'(x) = x^{\cos(2x)} \left[-\sin(2x) 2 \ln(x) + \frac{1}{x} \cos(2x) \right]$
- k. $y'(x) = (\tan(x))^{2x} \left[2 \left(\ln(\tan(x)) + \frac{x \sin(x)}{\cos^2 x} \right) \right]$
- l. $y'(x) = \sin(x)^{\ln(x)} \left[\frac{\ln(\sin(x))}{x} + \ln(x) \cot(x) \right]$

Implicit Differentiation

Questions

1) Find y' :

a. $x^2 + y^2 = 1$

b. $x^2 y^3 = x + y^2$

c. $\frac{y^2 + x}{y^3 - 4x} = 1$

d. $\sqrt{y} + \sqrt{x} = 1$

e. $(y + 2)^3 = xy$

f. $e^x + e^y = 1$

g. $\ln x + \ln y = y$

h. $(\ln y)^2 + y \ln x = 1$

i. $\sin y + \cos x = y^2$

j. $x \tan y = \sqrt{y}$

k. $x^y + y^x = 1$

l. $y^{\ln x} + x^{\ln y} = 4$

2) Find the first and the second derivatives:

a. $x^2 + y^3 = 1$

b. $x^2 y^3 = x + y$

c. $\ln x + \ln y = 1$

d. $\sin x + \sin y = x$

Answer Key

1) a. $y'(x) = -\frac{x}{y}$

b. $y'(x) = \frac{1-2xy^3}{3xy^2-2y}$

c. $y'(x) = \frac{-y^3-4y^2}{-y^4-8xy-3xy^2}$

d. $y'(x) = -\sqrt{\frac{y}{x}}$

e. $y'(x) = \frac{y}{3(y+2)^2-x}$

f. $y'(x) = -e^{x-y}$

g. $y'(x) = \frac{y}{x(y-1)}$

h. $y'(x) = -\frac{y^2}{z(2\ln y + y \ln x)}$

i. $y'(x) = \frac{\sin x}{\cos y - 2y}$

j. $y'(x) = \frac{\tan y}{\frac{1}{3\sqrt[3]{y^2}} - \frac{x}{\cos^2 y}}$

k. $y'(x) = -\frac{y^x(\ln y + 1)}{x^y \left(\ln x + \frac{x}{y} \right)}$

l. $y'(x) = -\frac{y \ln y}{x \ln x}$

2) a. $y'(x) = \frac{-2x}{3y^2}, y''(x) = \frac{2y^2 + 8x^2}{9y^6}$

b. $y'(x) = \frac{1-2xy^3}{3x^2y^2-1}, y''(x) = -2 \left[y^3 + 3xy^2 \frac{1-2xy^3}{3x^2y^2-1} \right] [3x^2y^2-1] - 6(1-2xy^3) \left(xy^2 + 2x^2y \frac{1-2xy^2}{3x^2y^2-1} \right)$

c. $y'(x) = -\frac{y}{x}, y''(x) = \frac{2y}{x^2}$

d. $y'(x) = \frac{1-\cos(x)}{\cos(y)}, y''(x) = \frac{(1+\sin(x))\cos(y)}{\sin(y)(\cos(x)-1)}$

Calculations Using the Definition of Derivative

Questions

1) Find the first derivative of the given function using the definition of the derivative:

a. $f(x) = x^2$

b. $g(x) = x^2 + 4x + 1$

c. $f(x) = x^3$

d. $y = \frac{1}{x}$

e. $f(x) = \sqrt{x}$

f. $z(x) = \ln x$

g. $y = e^x$

h. $y = \sin 2x$

Answer Key

1) a. $y'(x) = 2x$

d. $y'(x) = \frac{-1}{x^2}$

g. $y'(x) = e^x$

b. $y'(x) = 2x + 4$

e. $y'(x) = \frac{1}{2\sqrt{x}}$

h. $y'(x) = 2\cos(x)$

c. $y'(x) = 3x^2$

f. $y'(x) = \frac{1}{x}$

The Derivative of an Inverse of a Function

Questions

- 1) Prove that $(\ln x)' = \frac{1}{x}$. Use the rule of derivative of the inverse function.
- 2) Prove that $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$. Use the rule of derivative of the inverse function.
- 3) Prove that $(\arcsin x)' = \frac{1}{\sqrt{1+x^2}}$. Use the rule of derivative of the inverse function.
- 4) Prove that $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$. Use the rule of derivative of the inverse function.
- 5) Supposed that f^{-1} is the inverse function of a differentiable function f and that:
 $f(2) = 7$, $f'(2) = \sqrt{7}$. Find $(f^{-1})'(7)$.

Answer Key

1-4) Refer to the videos.

5) $\frac{1}{\sqrt{7}}$

Logarithmic Differentiation

Questions

- 1) Given the following function: $y = \sqrt[4]{\frac{10x-1}{x+1}} \cdot \sqrt[10]{(2x+1)^7}$, find y' .
- 2) Given the following function: $y = \left(\sqrt[4]{10x+1}\right)^{2x}$, find y' .
- 3) Find the equation of the line that is tangent to the curve: $f(x) = x^3 - 4x^2 + 2x - 5$
At the point on the curve where $x = 1$.
Does the line intersect the curve at any other point?

Answer Key

1) $y' = y \left(\frac{1}{4} \frac{10}{10x-1} - \frac{1}{4} \times \frac{1}{x+1} + \frac{7}{10} \frac{2}{2x+1} \right)$

2) $y' = \frac{1}{4} 2^x y \left[\ln 2 \ln(10x+1) + \frac{10}{10x+1} \right]$

3) $y = -3x - 3$, no.