proprep

Workbook

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Vectors

Introduction to 2D and 3D Vectors

Questions

1) Find each of the following 2D/3D vectors, its magnitude, and whether it's a unit vector:

a. The displacement vector from $(-8,3)$ to $(5,-2)$.

b. The displacement vector from $(2,3,4)$ to $(2,4,4)$.

c. The position vector for $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

d. The position vector for $(-8,3,5)$.

e. The vector $\vec{v}=\langle 7,-3,0\rangle$ starts at point $\mathrm{P}(-3,4,-1)$. At what point does it end?

Answer Key

1) a. $(13,-5)$ not unit vector b. $(0,1,0)$ is unit vector c. $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ is unit vector

d. $(-8,3,5)$ e. $(4,1,-1)$

Vector Arithmetic

Questions

1) Given the 2D vectors $\vec{a} = \langle 7, 4 \rangle$ and $\vec{b} = \langle -2, 5 \rangle$, compute the following:

a. $5\vec{a}$

b. $6\vec{b} - 3\vec{a}$

c. $\left\| 9\vec{a} + 4\vec{b} \right\|$

2) Given the 3D vectors $\vec{u} = 7\vec{i} - 2\vec{j} + 4\vec{k}$ and $\vec{v} = 6\vec{j} - 2\vec{k}$, compute the following:

a. $-4\vec{v}$

b. $10\vec{u} + \vec{v}$

c. $\| -8\vec{u} - 3\vec{v} \|$

3) Answer the following questions:

a. Find a unit vector that points in the same direction as $\vec{v} = \vec{i} - 4\vec{j} + 8\vec{k}$.

b. Find a vector that points in the same direction as $\vec{w} = \langle -2, 5 \rangle$ with a magnitude of 10.

4) Determine if the following pairs of vectors are parallel.

a. $\vec{v} = 6\vec{i} - 4\vec{j} - 16\vec{k}$ and $\vec{w} = 15\vec{i} - 10\vec{j} - 40\vec{k}$.

b. $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 6, -4, 7 \rangle$.

Answer Key

1) a. $(35, 20)$ b. $(-33, 18)$ c. $\sqrt{6161} = 78.492$

2) a. $-24\vec{j} + 8\vec{k}$ b. $70\vec{i} - 14\vec{j} + 38\vec{k}$ c. $\sqrt{3816} = 61.77$

3) a. $\vec{v} = \frac{1}{9}\vec{i} - \frac{4}{9}\vec{j} + \frac{8}{9}\vec{k}$ b. $\left(-\frac{20}{\sqrt{29}}, \frac{50}{\sqrt{29}}\right)$

4) proof.

Vectors Dot Product

Questions

1) In each of the following, find the dot product $\vec{a}\cdot\vec{b}$:

a. $\vec{a}=\langle 5,-4\rangle$ and $\vec{b}=\langle 4,3\rangle$.

b. $\vec{a}=8\vec{i}+6\vec{j}-3\vec{k}$ and $\vec{b}=6\vec{i}-4\vec{j}+7\vec{k}$.

c. $\|\vec{a}\|=4$, $\|\vec{b}\|=3$, and the angle between the two vectors is $\frac{\pi}{3}$.

2) In each of the following, find the angle between the two vectors:

a. $\vec{a}=\langle 3,5\rangle$, $\vec{b}=\langle 7,6\rangle$.

b. $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{w}=5\vec{i}+6\vec{j}-7\vec{k}$.

3) In each of the following, determine if the two vectors are parallel, orthogonal or neither:

a. $\vec{p}=\langle 1,-2,3\rangle$, $\vec{q}=\langle 5,-8,-7\rangle$.

b. $\vec{a}=\langle 3,5\rangle$, $\vec{b}=\langle 7,6\rangle$.

c. $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{w}=-5\vec{i}+10\vec{j}-15\vec{k}$.

4) Compute $proj_{\vec{u}}\vec{v}$ [projection of $\vec{v}$ onto $\vec{u}$] for the following pairs of 2D/3D vectors:

a. $\vec{u}=\langle 4,-1\rangle$, $\vec{v}=\langle 1,7\rangle$.

b. $\vec{u}=7\vec{i}-\vec{j}+\vec{k}$, $\vec{v}=-2\vec{i}+5\vec{j}-6\vec{k}$.

5) Find the direction cosines and direction angles for $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$.

Answer Key

1) a. 8 b. 3 c. 6

2) a. $\theta = 18.435°$ b. $\theta = 135.521°$

3) a. Orthogonal b. Neither c. Parallel

4) a. $\left(-\frac{12}{17}, \frac{3}{17}\right)$ b. $\left(-\frac{161}{51}, \frac{23}{51}, -\frac{23}{51}\right)$

5) Solution is in the video.

Vectors Cross Product

Questions

1) Answer the following questions:

a. Given $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 6, -4, 7 \rangle$. Compute $\vec{a} \times \vec{b}$ and use the result to find $\vec{b} \times \vec{a}$.

b. Given $\vec{u} = 3\vec{i} - \vec{j} + 5\vec{k}$ and $\vec{v} = 4\vec{j} - 2\vec{k}$. Compute $\vec{u} \times \vec{v}$ and use the result to find $\vec{v} \times \vec{u}$.

2) Find a vector that is orthogonal to the plane containing the points $\mathrm{P}(1,2,3)$, $\mathrm{Q}(6,5,4)$ and $\mathrm{R}(7,8,9)$.

3) For each of the following sets of 3 vectors, determine if they lie in the same plane or not:

a. $\vec{a} = \langle 3, -2, 5 \rangle$, $\vec{b} = \langle 6, -4, 7 \rangle$, $\vec{c} = \langle 1, 0, 1 \rangle$.

b. $\vec{u} = \vec{i} + 4\vec{j} - 7\vec{k}$, $\vec{v} = 2\vec{i} - \vec{j} + 4\vec{k}$, $\vec{w} = -9\vec{j} + 18\vec{k}$.

Answer Key

1) a. $(6,9,0),(-6,-9,0)$ b. $-18\vec{i}+6\vec{j}+12\vec{k}$, $18\vec{i}-6\vec{j}-12\vec{k}$

2) $\langle 12,-24,12\rangle$

3) a. Not in the same plane. b. In the same plane

The 3D Coordinates System

Questions

1) Find the projection of the point $\mathrm{P}(4,7,-5)$ onto the three coordinate planes.

2) Answer the following questions:

a. What is the distance of the point $\mathrm{P}(4,7,-5)$ from the z-axis?

b. Which point is closer to the z-axis: $\mathrm{P}(4,7,-5)$ or $\mathrm{Q}(5,-6,7)$?

3) Answer the following questions:

a. What is the distance of the point $\mathrm{P}(4,7,-5)$ from the xy-plane?

b. Which point is closer to the xy-plane: $\mathrm{P}(4,7,-5)$ or $\mathrm{Q}(5,-6,7)$?

4) Answer the following questions:

a. Given the equation $2x+3y=6$,

i. In $\mathbb{R}^2$ this is the equation of a:_____

ii. In $\mathbb{R}^3$ this is the equation of a:_____

b. Given the equation $(x-1)^2+y^2=9$,

i. In $\mathbb{R}^2$ this is the equation of a: _____

ii. In $\mathbb{R}^3$ this is the equation of a: _____

Answer Key

1) Onto xy: $(4,7,0)$. Onto xz: $(4,0,-5)$. Onto yz: $(0,7,-5)$.

2) a. 5 b. Point P is closer.

3) a. $\sqrt{65}$ b. Point Q is closer.

4) a.(i) Line a.(ii) plane b.(i) Circle b.(ii) Cylinder.

Equations of Lines

Questions

1) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the points $(-10,4,0)$ and $(1,-4,2)$.

2) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the point $(-10,4,0)$ and is parallel to the line $x=3+4t$, $y=-2+3t$, $z=-5t$.

3) Let l_1 be the line through points $(4,1,-5)$ and $(2,0,9)$ and let l_2 be the line given by $\vec{r}(t)=\langle 5,1-9t,-8-4t\rangle$. Are the lines l_1 and l_2 parallel, perpendicular or neither?

4) Let l_1 be the line given by $x=-7+12t$, $y=3-t$, $z=14+8t$ and let l_2 be the line given by $\vec{r}(t)=\langle 8+t,5+6t,4-2t\rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.

5) Let l_1 be the line passing through points $(-5,0,2)$ and $(13,-2,1)$ and let l_2 be the line given by $\vec{r}(t)=\langle 3,-1-t,2+4t\rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.

6) Let l be the line given by $x=-7+12t$, $y=3$, $z=16+8t$.

a. Does l intersect the xy-plane? If so, where?

b. Does l intersect the xz-plane? If so, where?

Answer Key

1) $\langle -10+11t, 4-8t, 2t \rangle$ $x=-10+11t$, $y=4-8t$, $z=-2t$ $\frac{x+10}{11}-\frac{y-4}{8}=\frac{z}{2}$

2) $\langle -10+4t, 4+3t, -5t \rangle$ $t=\frac{x+10}{4}$, $t=\frac{y-4}{3}$, $t=-\frac{z}{5}$ $\frac{x+10}{4}=\frac{y-4}{3}=-\frac{z}{5}$

3) Perpendicular

4) $v=\frac{-39}{73}$; $u=\frac{88}{73}$

5) $\left(3,-\frac{8}{9},1\frac{5}{9}\right)$

6) a. $(-31,3,0)$ b. Not intersection.

Equations of Planes

Questions

1) Find the equation of the plane through the points $\mathrm{P}(0,1,1)$, $\mathrm{Q}(1,0,1)$ and $\mathrm{R}(1,-3,-1)$.

2) Find the equation of the plane passing through the point $(0,2,-1)$ and orthogonal to the line $\vec{r}(t)=\langle 5+t,1+3t,4t\rangle$.

3) Find the equation of the plane containing the point $(-7,3,9)$ and parallel to the plane $4x+8y-2z=37$.

4) Plane π_1 is given by $4x+8y-2z=10$ and plane π_2 is given by $2x+y+8z=11$.
Are the planes π_1 and π_2 parallel, orthogonal or neither?

5) Plane π_1 is given by $2x-3y+4z=5$ and plane π_2 passes though points $(1,2,2)$, $(2,2,3)$ and $(-3,-2,-6)$. Are the planes π_1 and π_2 parallel, orthogonal or neither?

6) Plane π is given by $2x-y+3z=6$ and line l is given by $x=1-t$, $y=3t$, $z=1+t$.
Do l and π intersect? If so, where?

7) Plane π is given by $x-y+z=3$ and line *l* is given by $\vec{r}(t)=\langle 5+2t,1-5t,3t\rangle$.
Do l and π intersect? If so, where?

8) Two planes π_1 and π_2 are given by $-x+7y-2z=24$ and $-5x+6y+3z=-3$, respectively.
l is the line intersection of the planes: $l=\pi_1\cap\pi_2$. Find the vector equation of line l.

9) Plane π is given by $5x-3y-6z=4$ and line l is given by $\vec{r}(t)=\langle 5-10t,1+6t,12t\rangle$.
Are l and π parallel, perpendicular or neither?

Answer Key

1) $2x+2y-3z=-1$

2) $x+3y+4z=2$

3) $4x+8y-2z=-22$

4) Orthogonal.

5) Neither.

6) $\left(1\frac{1}{2},-1\frac{1}{2},\frac{1}{2}\right)$

7) Not intersect.

8) $\vec{r}(t)=\langle 33t,2+13t,-5+29t\rangle$

9) Perpendicular.

Quadric Surfaces

Questions

1) Sketch the graph of the quadric surface $\frac{y^2}{9}+z^2=1$.

2) Sketch the graph of the quadric surface $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{6}=1.$

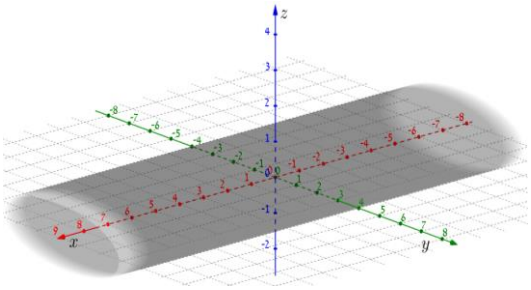
3) Sketch the graph of the quadric surface $z=\frac{x^2}{4}+\frac{y^2}{4}-6$.

4) Sketch the graph of the quadric surface $y^2=4x^2+16z^2$.

5) Sketch the graph of the quadric surface $x=4-5y^2-9z^2$.

Answer Key

1)



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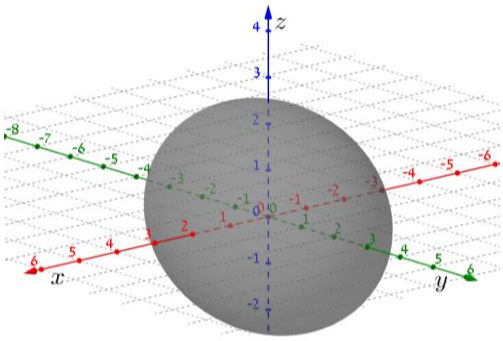

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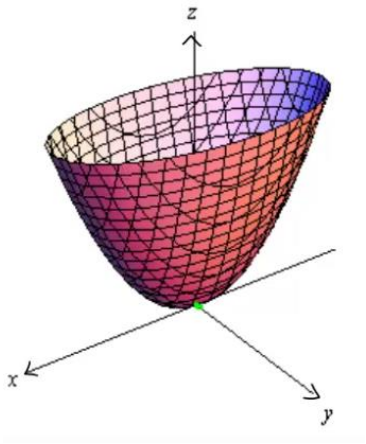

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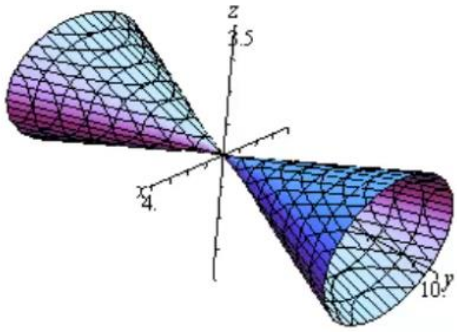

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