proprep

Workbook

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Energy Integrals and Uniqueness of Solutions

Energy Integrals and Uniqueness of Solutions

Questions

1) Use an energy integral to show uniqueness of the solution to the following IBVP:

$$\begin{cases} u_{tt} = u_{xx} + F(x,t) & 0 \le x \le L, \quad t \ge 0 \\ u(x,0) = f(x) & u_t(x,0) = g(x) \\ u_x(0,t) = A(t) & u_x(L,t) = B(t) \end{cases}$$

Hint: define $E(t) = \frac{1}{2}\int_0^L \left[(w_t)^2 + (w_x)^2 \right] dx$ for suitable w.

2) Use an energy integral to show uniqueness of the solution to the following IBVP:

$$\begin{cases} u_{tt} = u_{xx} - u + F(x,t) & 0 \le x \le L, \quad t \ge 0 \\ u(x,0) = f(x) & u_t(x,0) = g(x) \\ u_x(0,t) = A(t) & u_x(L,t) = B(t) \end{cases}$$

Hint: define $E(t) = \frac{1}{2}\int_0^L \left[(w_t)^2 + (w_x)^2 + w^2 \right] dx$.

3) Use an energy integral to show uniqueness of the solution to the following IBVP:

$$\begin{cases} u_t = u_{xx} + F(x,t) & 0 \le x \le L, \quad t \ge 0 \\ u(x,0) = f(x) & \\ u(0,t) = A(t) \qquad u(L,t) = B(t) & \end{cases}$$

Hint: define $E(t) = \frac{1}{2}\int_0^L w^2(x,t)dx$.

4) Use an energy integral to show uniqueness of the solution to the following IBVP:

$$\begin{cases} u_t = u_{xx} + F(x,t) & 0 \le x \le L, \quad t \ge 0 \\ u(x,0) = f(x) & \\ u_x(0,t) = A(t) \qquad u_x(L,t) = B(t) & \end{cases}$$

Hint: define $E(t) = \frac{1}{2}\int_0^L \left[w_x^{\ 2}(x,t) + w^2(x,t) \right] dx$.

5) Use an energy integral to show uniqueness of the solution to the following IBVP:

$$\begin{cases} u_t = u_{xx} - \beta u + F(x,t) & 0 \le x \le L, \quad t \ge 0, \quad \beta > 0 \\ u(x,0) = f(x) & \\ u_x(0,t) = A(t) \qquad u_x(L,t) = B(t) & \end{cases}$$

Hint: define $E(t) = \frac{1}{2}\int_0^L \left[w_x^{\ 2}(x,t) + w^2(x,t) \right] dx$.

Answer Key

1) Proof

2) Proof

3) Proof

4) Proof

5) Proof